Rotorcraft Unmanned Aerial Systems
An Introduction to Modeling and System Identification

Kostas Alexis & Christoph Huerzeler
3 July 2012

Autonomous Systems Lab
ETH Zürich

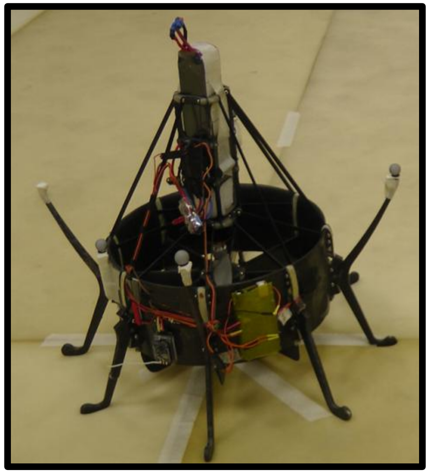
Lecture Overview

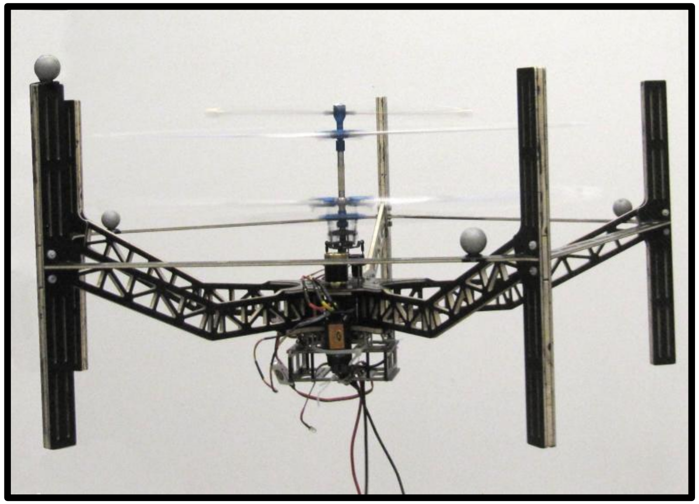
- Content
  - Introduction
  - Modeling Approaches & Challenges
  - Rotorcraft Modeling
  - Identification Techniques

- Goals
  - Provide Basic Introduction for Self-Study
  - Emphasize Core Challenges
  - Define Research Trends

Autonomous Systems Lab

Rotary Wing UAV Configurations



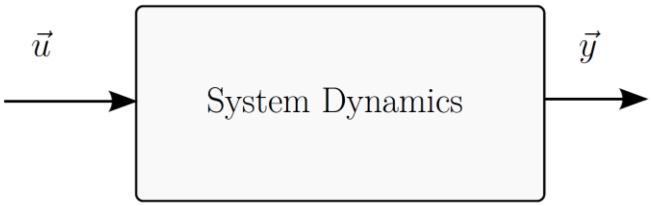


- Various Types of UAV Configuration Exist
- Underyling Modeling Approaches Similar
- Learn From Full-Scale Rotorcraft Theory

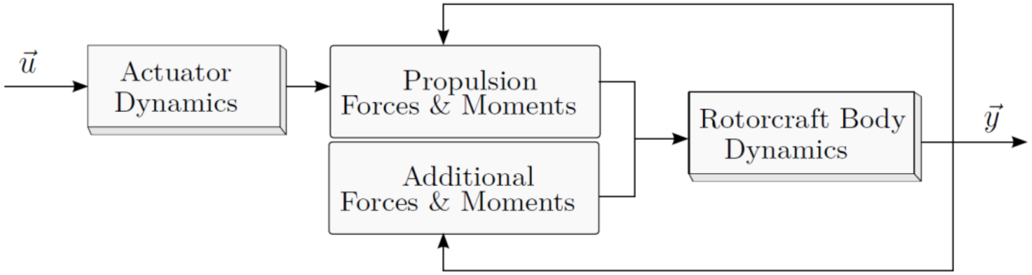
ETH Zürich

Modeling Approaches

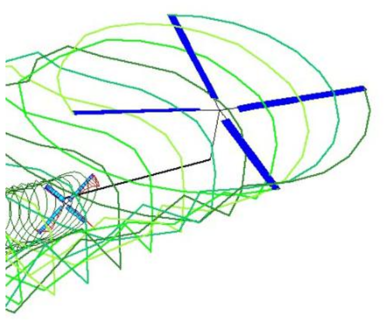
- Blackbox
  - Identify input-output behavior
  - No understanding of underlying system

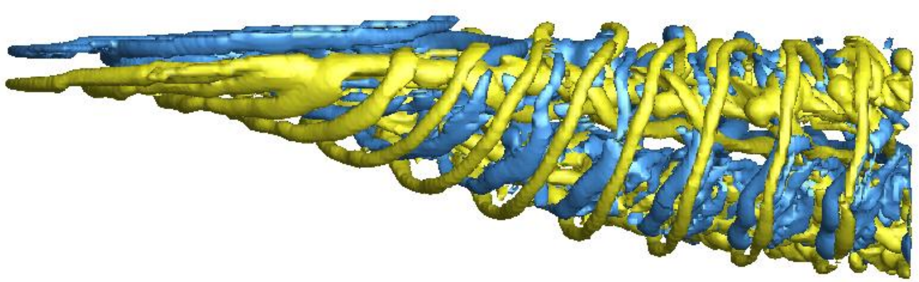


- Greybox
  - Based on laws of physics
  - Identify model parameters



- Whitebox
  - Purely based on laws of physics
  - All parameters predicted
  - Numerical or analytical

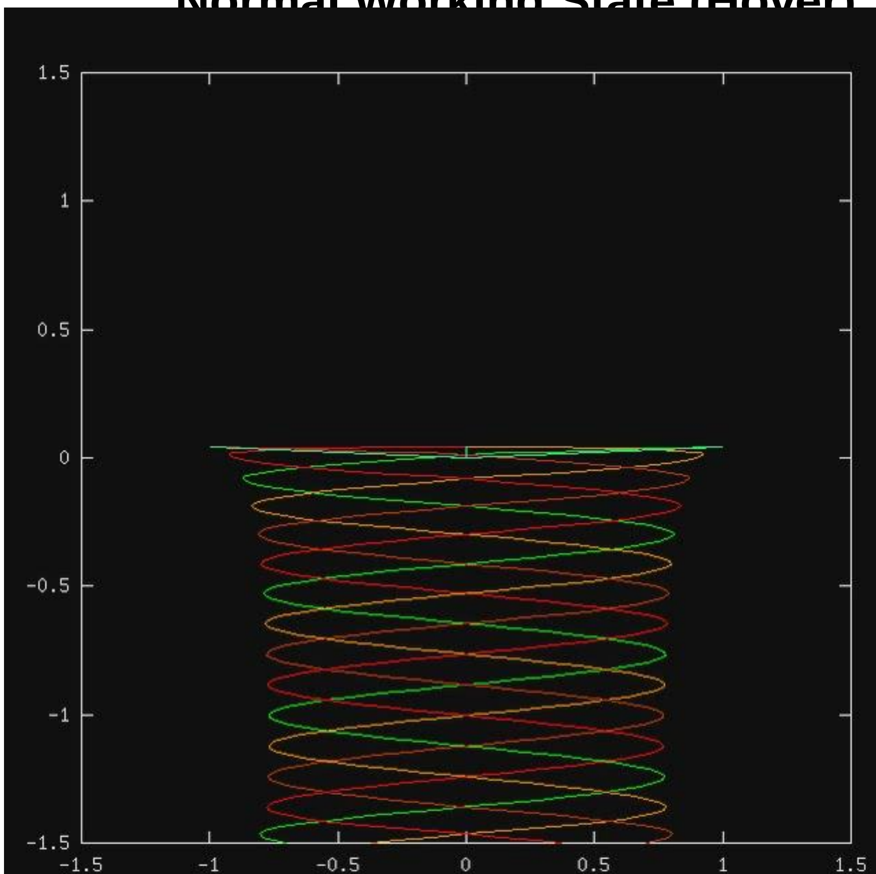




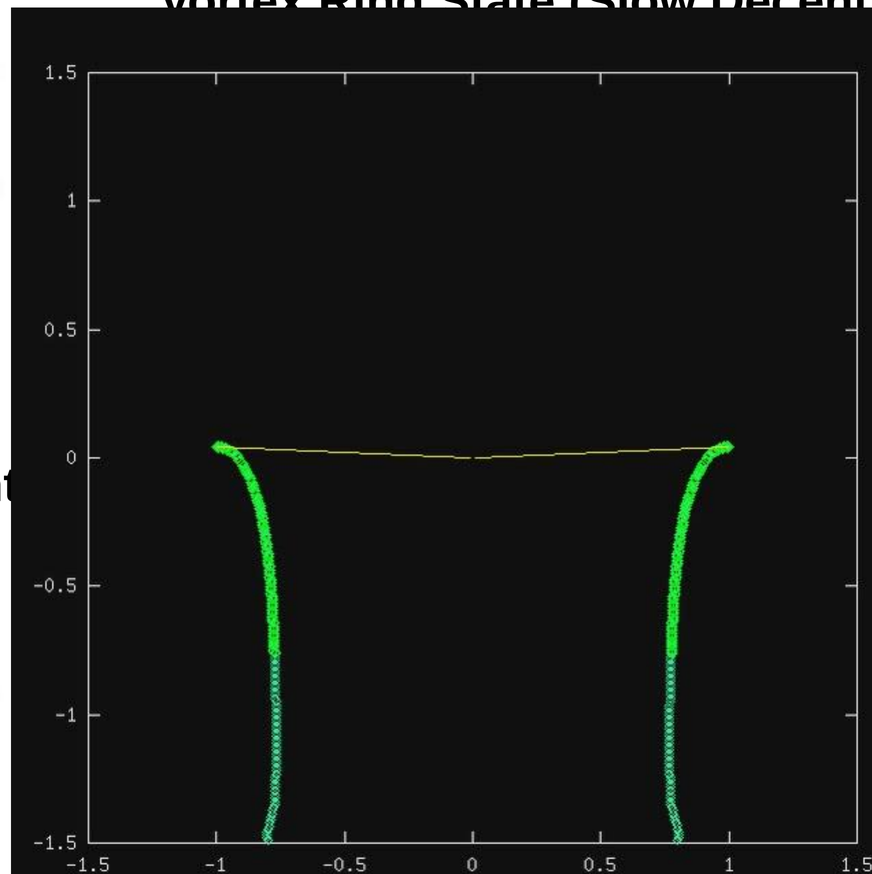
Modeling Challenges: Operation Enveloppe

- Axial Flight

Normal Working State (Hover)

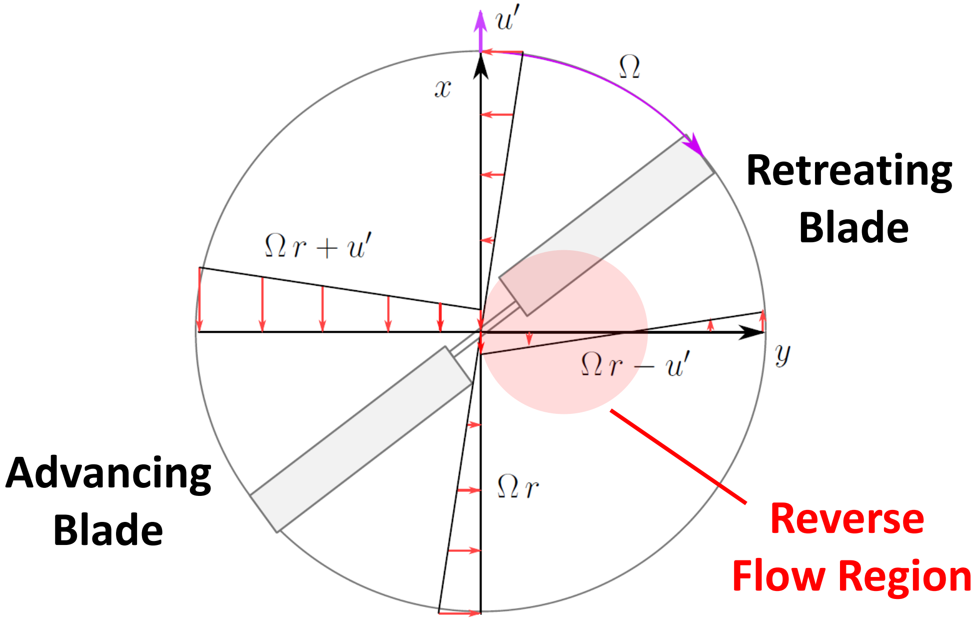


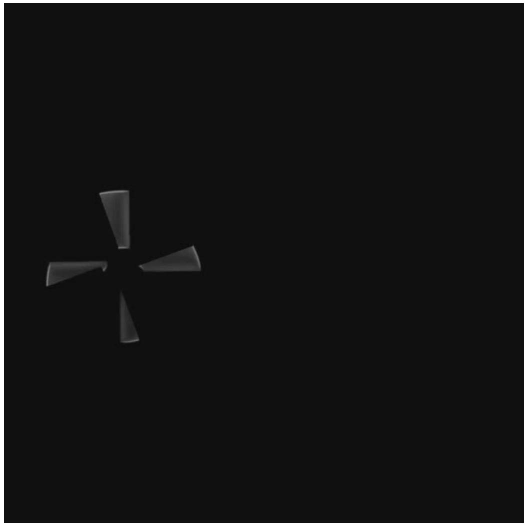
Vortex Ring State (Slow Decent)



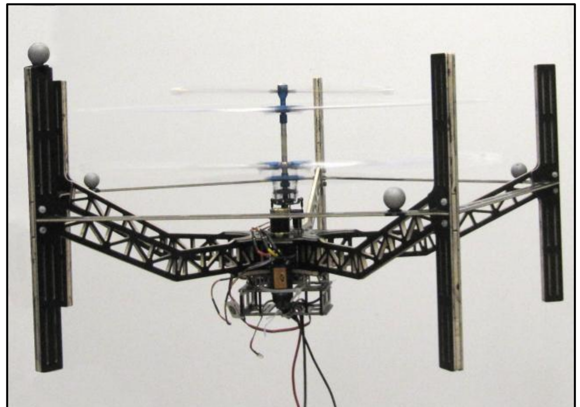
Modeling Challenges: Operation Enveloppe

- Axial Flight
- Forward Flight





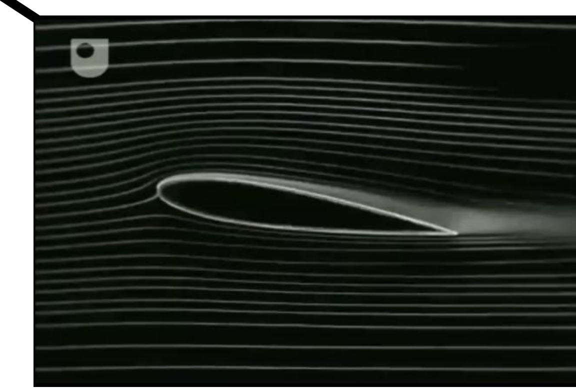
Modeling Challenges: Coupled Non-Linear Dynamics



AIRobots Coaxial (helicopter type UAV)

Main Body Motion («slow» dynamics)

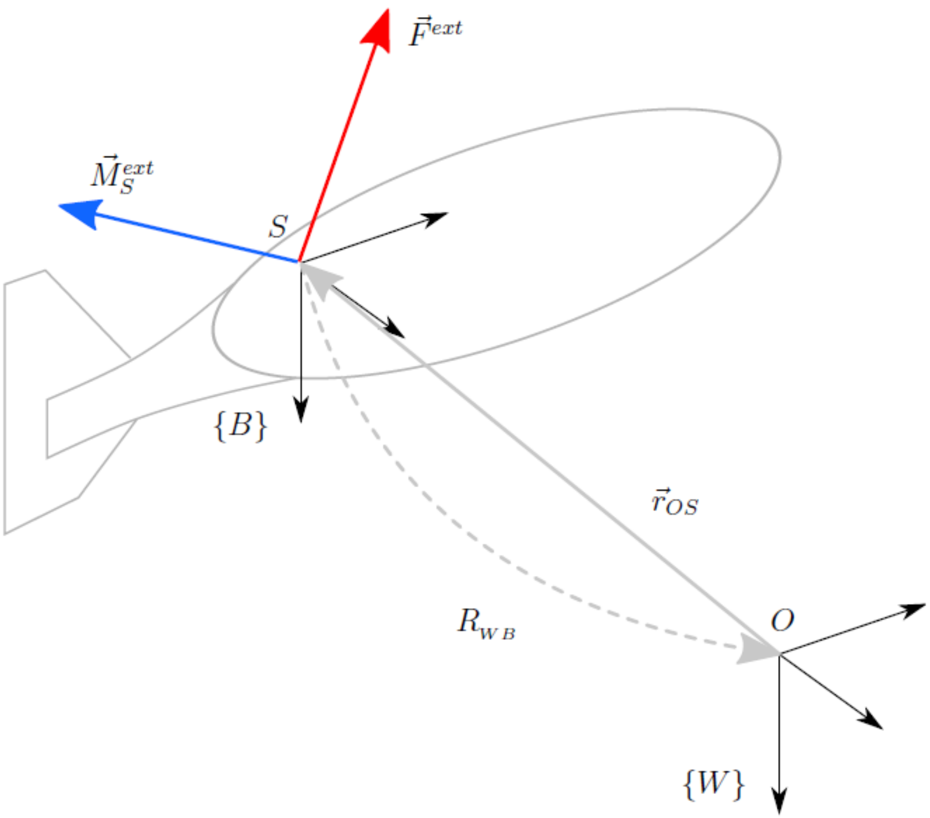
Rotorblade Motion («fast» dynamics)

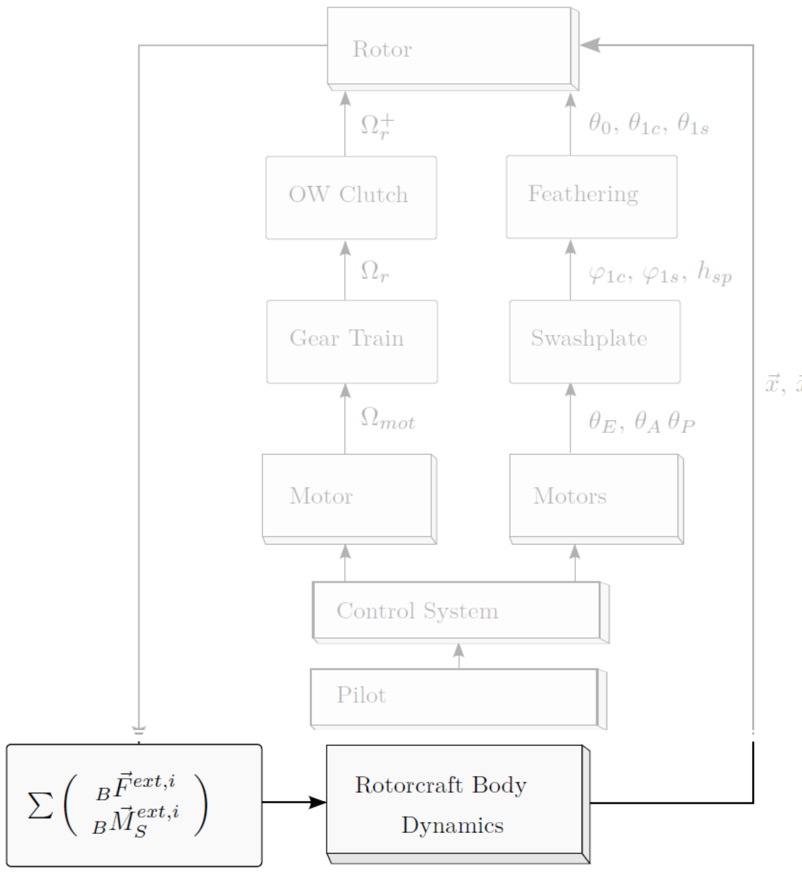


Rotorblade Aerodynamics

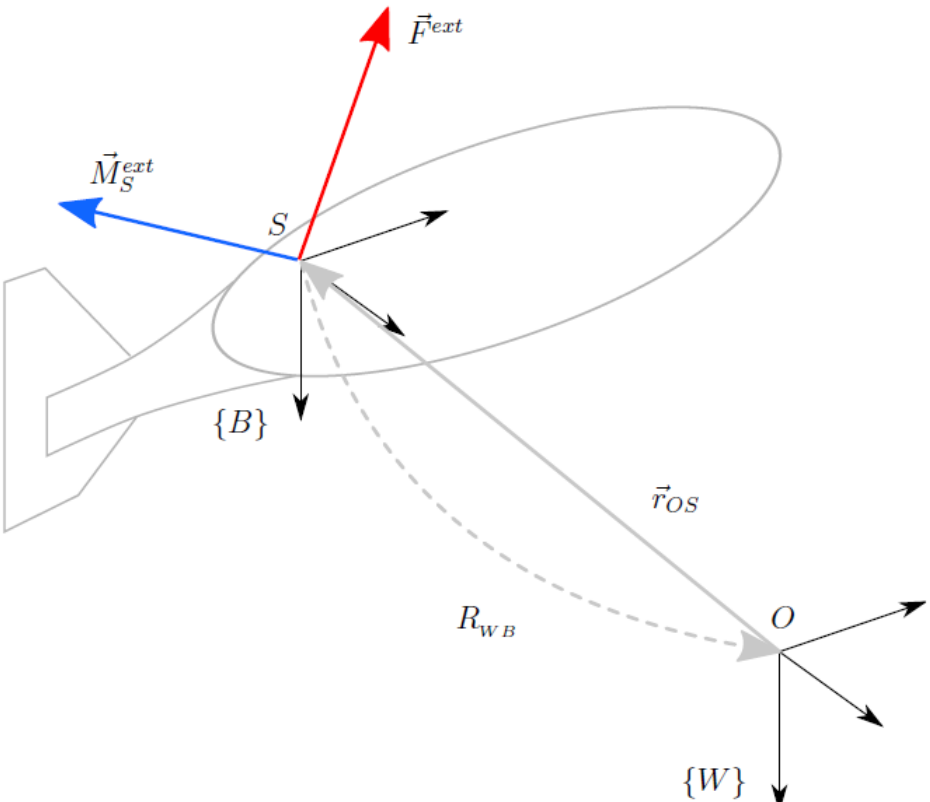
Source: open.edu/youtube

Rotorcraft Dynamics: An Overview





Rotorcraft Dynamics: An Overview



$$
{}_W\vec{r}_{OS} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad {}_W\dot{\vec{r}}_{OS} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}
$$

$$
{}_B\vec{v}_S = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad {}_B\dot{\vec{v}}_S = \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix}
$$

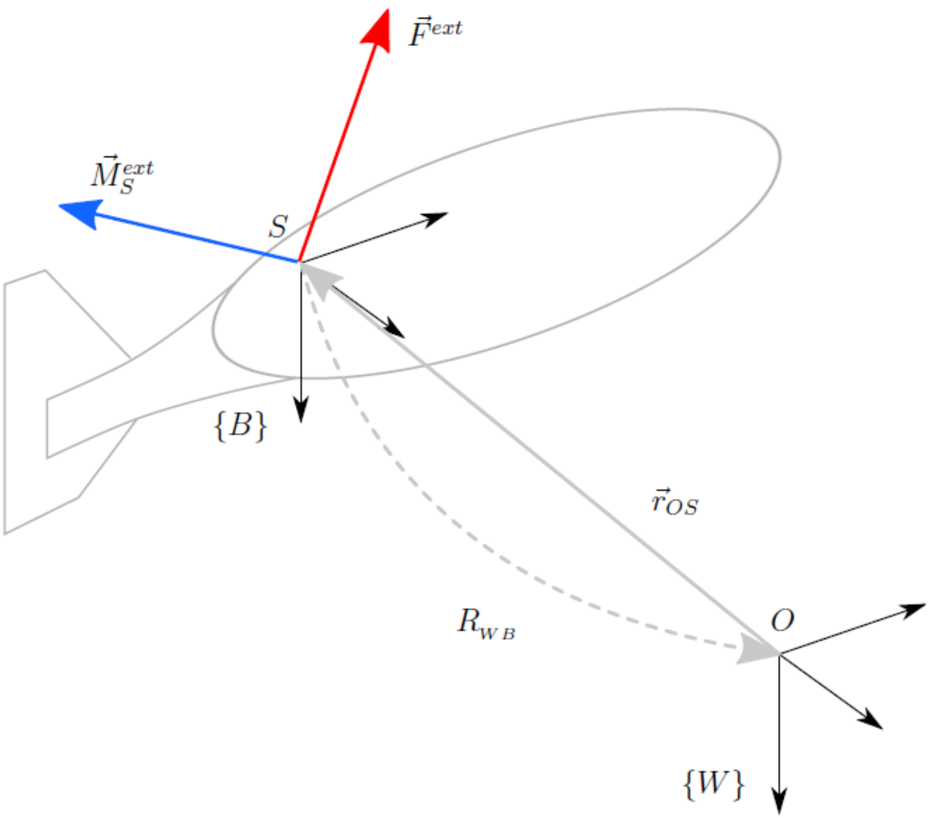
$$
{}_B\dot{\vec{\Omega}} = \begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} \quad {}_B\vec{\Omega} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}
$$

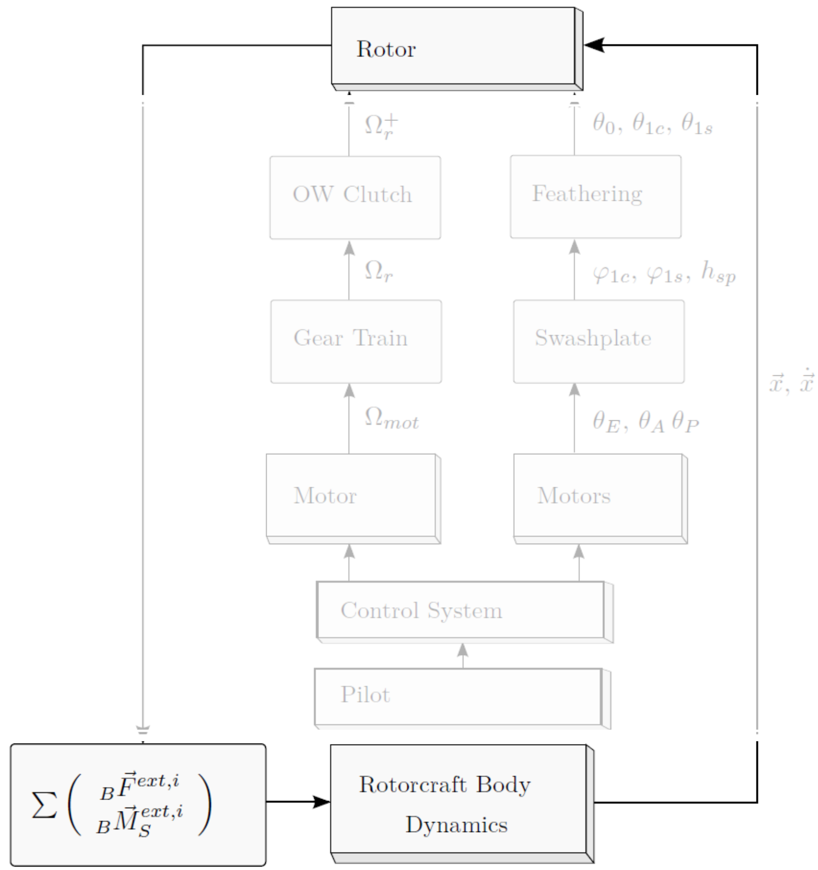
$$
\boxed{\begin{aligned} m\left({}_B\dot{\vec{v}}_S + {}_B\vec{\Omega} \times {}_B\vec{v}_S\right) &= {}_B\vec{F}^{ext} \\ {}_B\overline{\Theta}_S\, {}_B\dot{\vec{\Omega}} + {}_B\vec{\Omega} \times \left({}_B\overline{\Theta}_S\, {}_B\vec{\Omega}\right) &= {}_B\vec{M}_S^{ext} \end{aligned}}
$$

Momentum Conservation

Angular Momentum Conservation

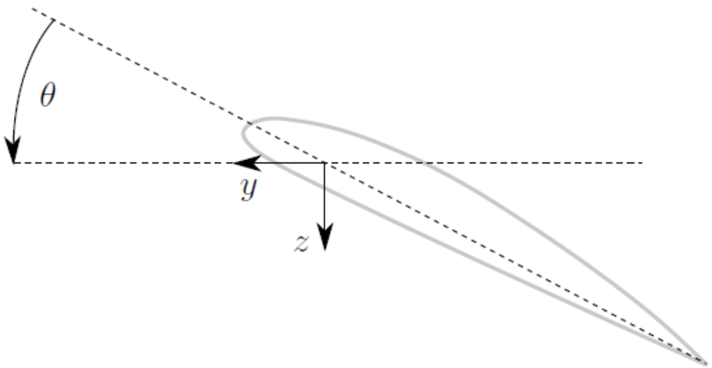
Rotorcraft Dynamics: Rotor DOF

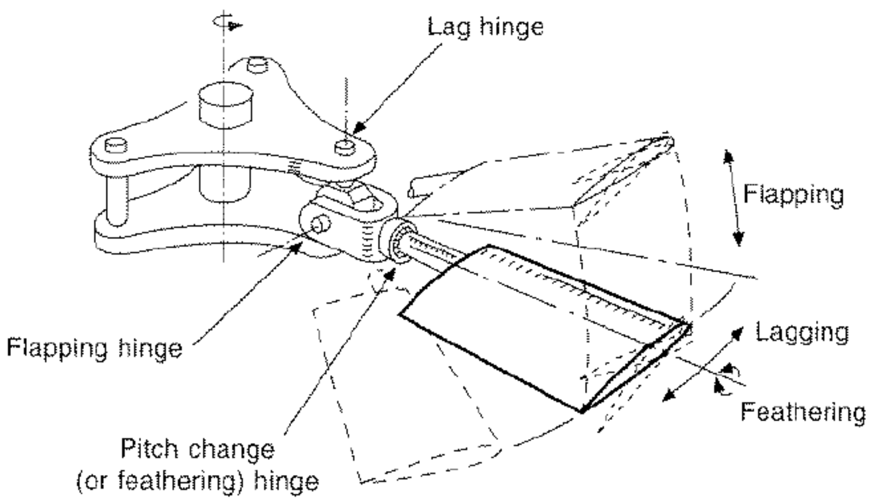




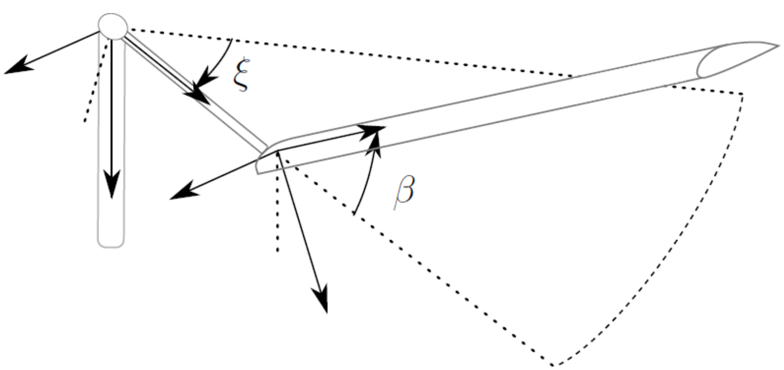
Rotorcraft Modeling: Rotor DOF

- Blade Feathering («Pitch»)



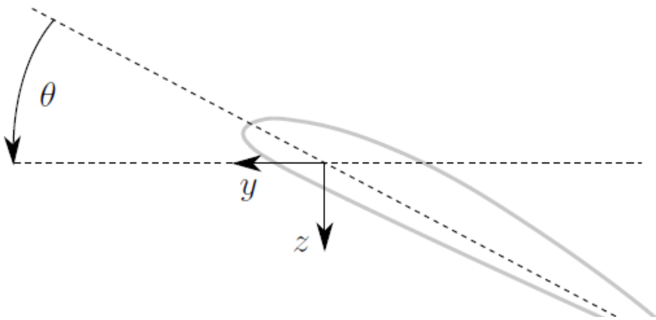


- Blade Flapping



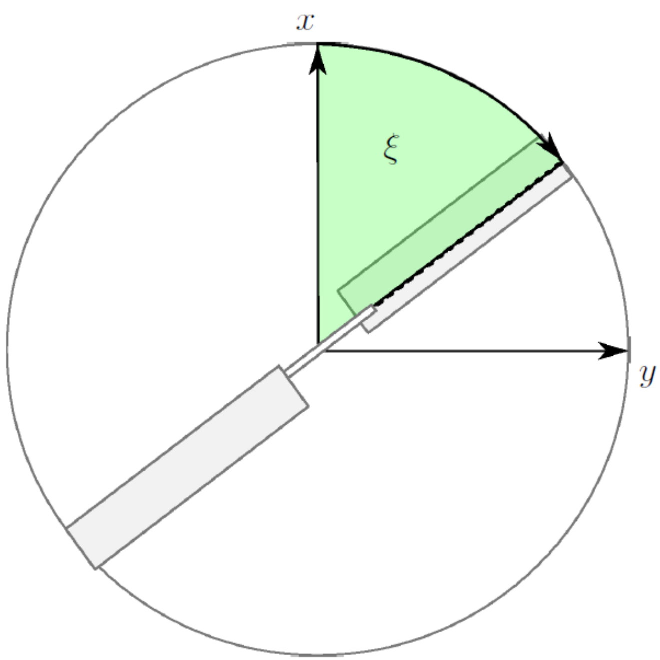
Rotorcraft Modeling: Rotor DOF

- Blade Feathering



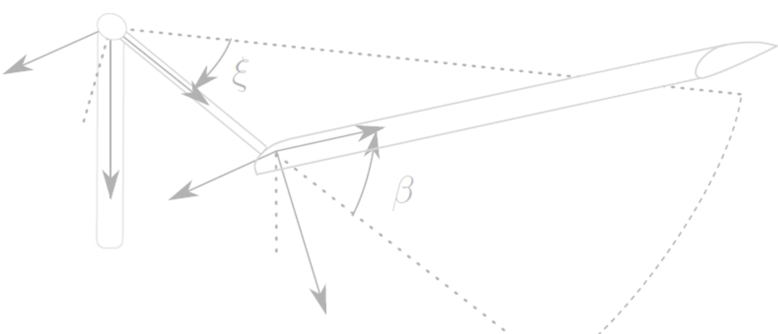
$$\theta(\xi) = \theta_0 + \theta_{1c}\, cos(\xi) + \theta_{1s}\, sin(\xi)$$

$\boxed{\theta_{1c}}$



$\boxed{\theta_{1s}}$

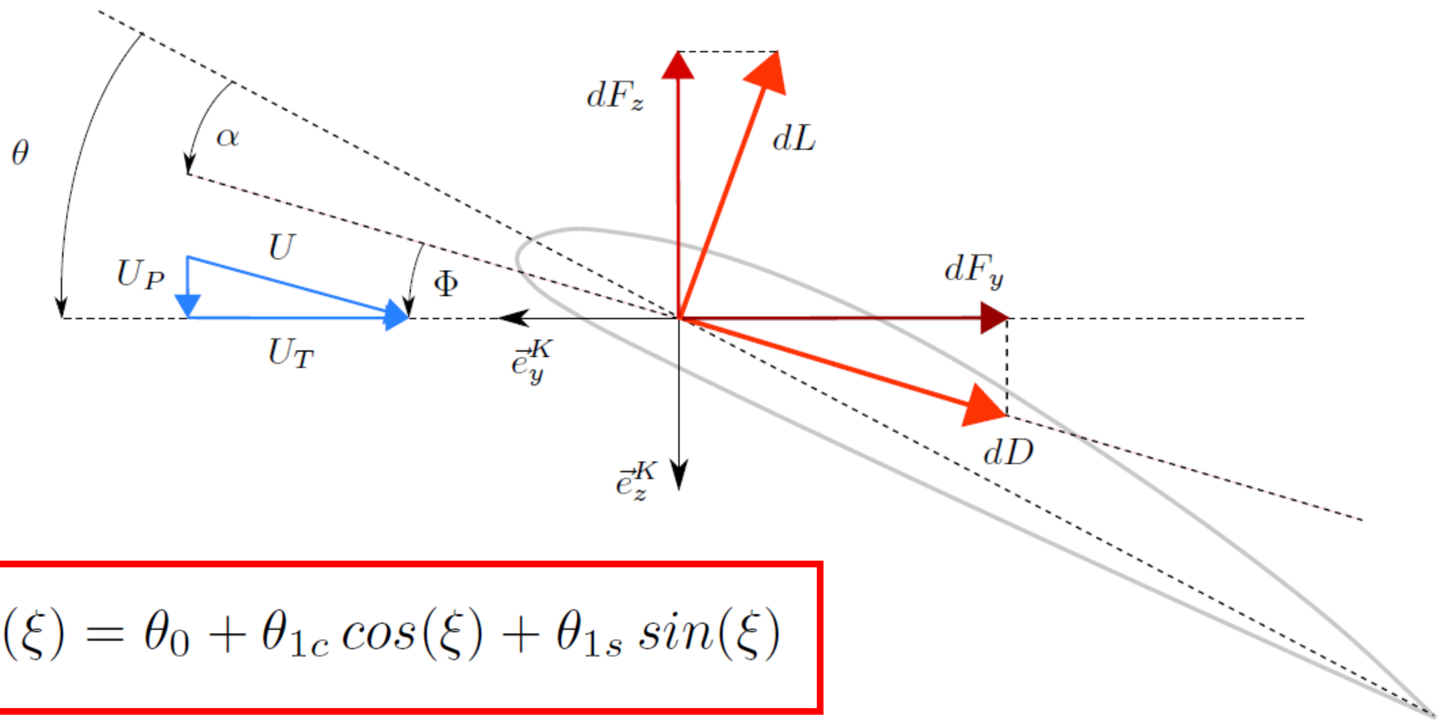
- Blade Flapping



$$\beta(\xi) = \beta_0 + \beta_{1c}\, cos(\xi) + \beta_{1s}\, sin(\xi)$$

Rotorcraft Modeling: Rotor Feathering

- Blade Element Theory



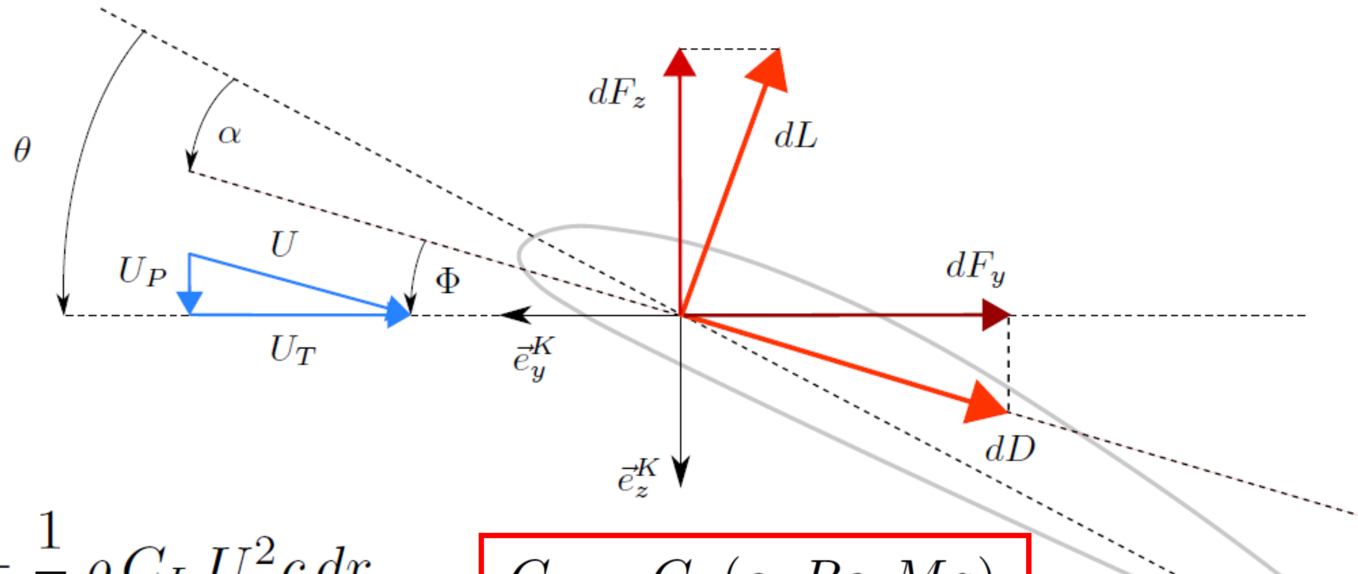
$$\boxed{\theta(\xi) = \theta_0 + \theta_{1c}\, cos(\xi) + \theta_{1s}\, sin(\xi)}$$

$$dF_y = dD\, cos(\phi) + dL\, sin(\phi) \approx dD + \phi\, dL$$ → **Rotor Torque**

$$dF_z = dL\, cos(\phi) - dD\, sin(\phi) \approx dL - \phi\, dD$$ → **Rotor Thrust**

Rotorcraft Modeling: Rotor Feathering

- Blade Element Theory



$$dL = \frac{1}{2}\rho C_L U^2 c\, dr$$

$$dD = \frac{1}{2}\rho C_D U^2 c\, dr$$

$$\boxed{\begin{aligned} C_L &= C_L(\alpha, Re, Ma) \\ C_D &= C_D(\alpha, Re, Ma) \end{aligned}}$$

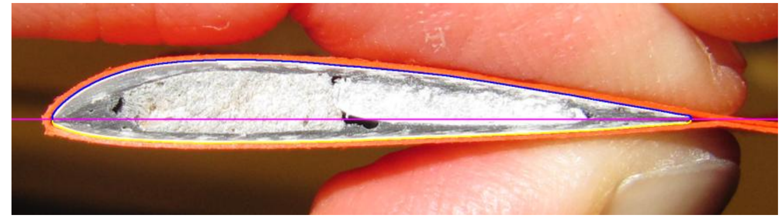
$$\alpha = \theta - \boxed{\phi}$$

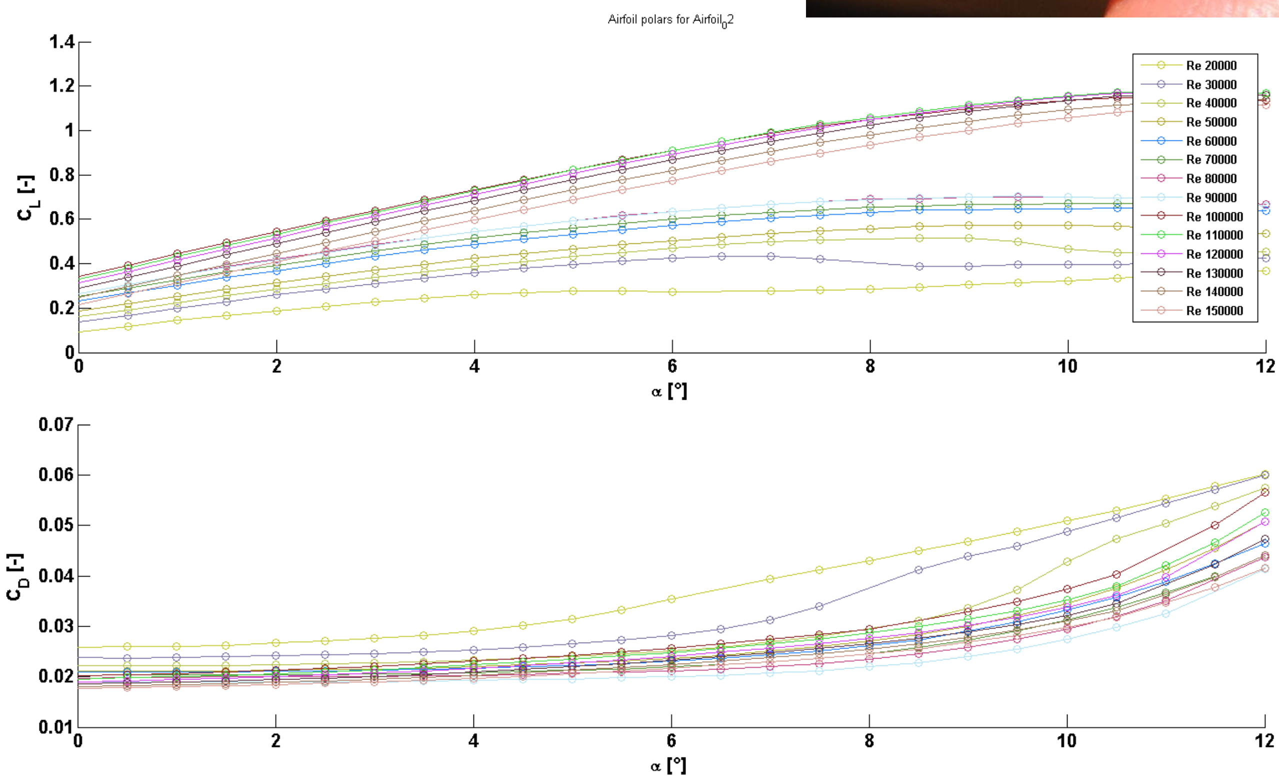
$$\boxed{\phi = atan(\frac{U_P}{U_T})}$$

$$\alpha = \theta - \boxed{\frac{U_P}{U_T}}$$

Rotorcraft Modeling: Rotor Feathering

- Aerodynamic Coefficients

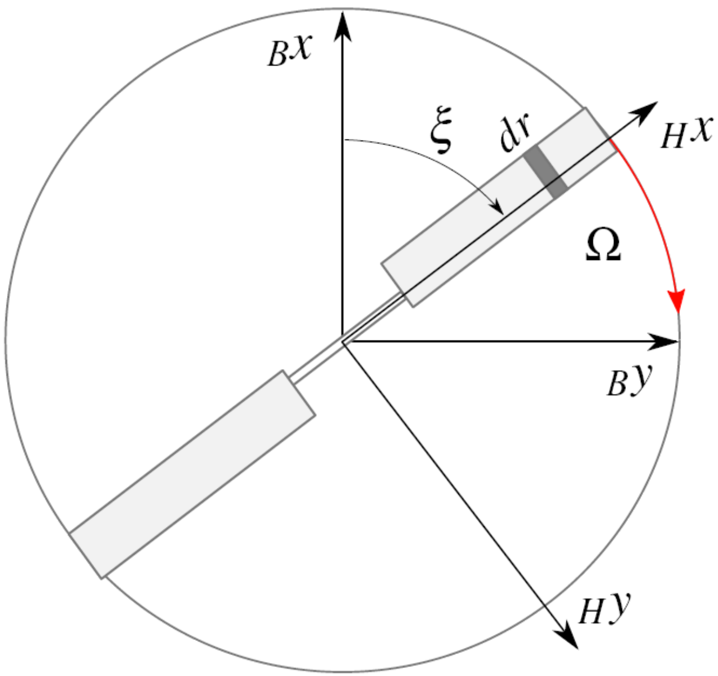


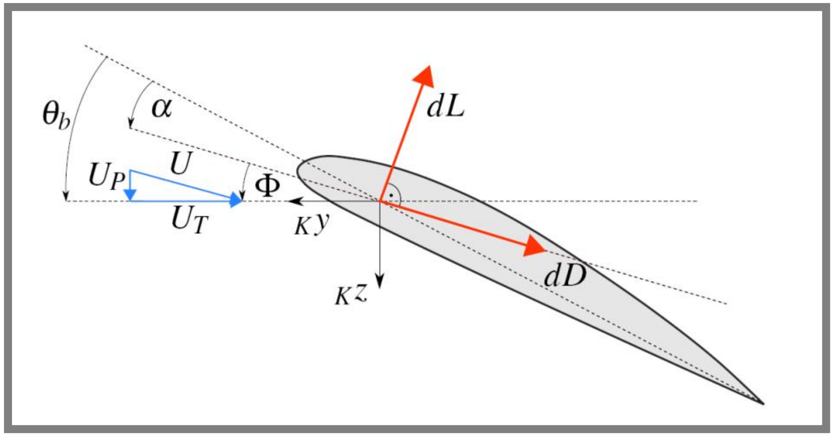


$$C_L = C_{L1}\,\alpha + C_{L0}$$

$$C_D = C_{D2}\,\alpha^2 + C_{D1}\,\alpha + C_{D0}$$

Rotorcraft Modeling: Thrust & Torque





$$\theta_b(\xi) = \theta_0 + \theta_{1c}\cos(\xi) + \theta_{1s}\sin(\xi)$$

$$T = \frac{N_b}{2\pi}\int_0^{2\pi}\int_0^R dL(r,\theta)$$

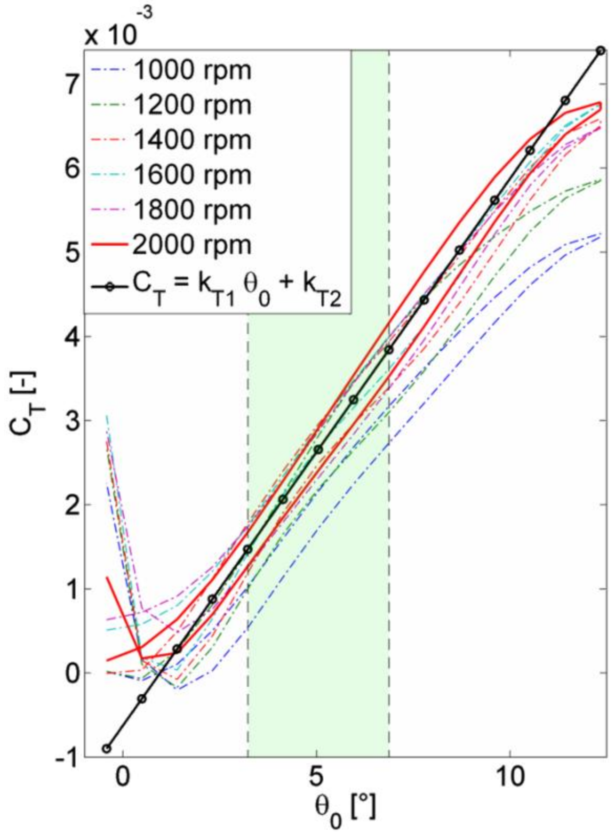
$$Q = \frac{N_b}{2\pi}\int_0^{2\pi}\int_0^R \left(dD(r,\theta) + \Phi\, dL(r,\theta)\right)$$

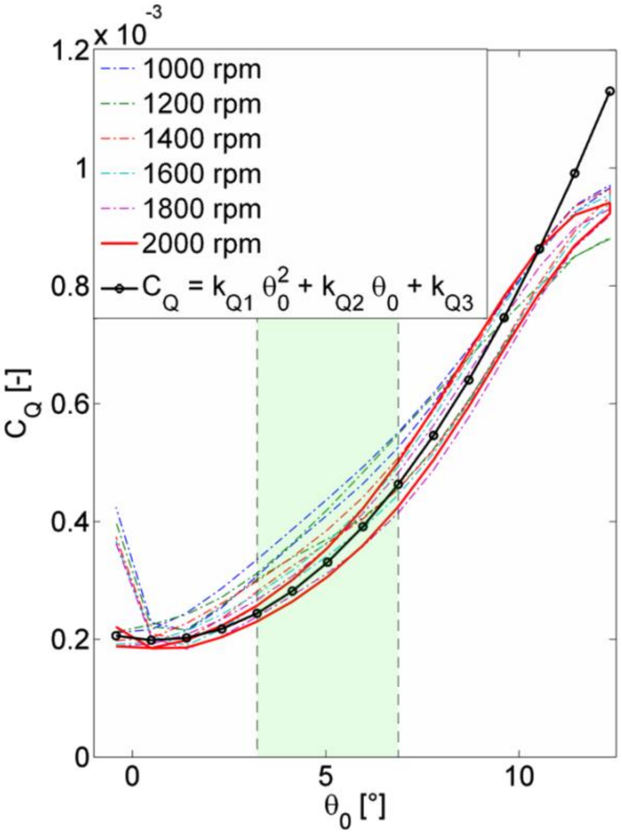
Averaged Thrust & Torque

Rotorcraft Modeling: Thrust & Torque

$$T = \left(k_{T_1}\,\theta_0 + k_{T_0}\right)\,\Omega^2$$

$$Q = \left(k_{Q_2}\,\theta_0^2 + k_{Q_1}\,\theta_0 + k_{Q_0}\right)\,\Omega^2$$

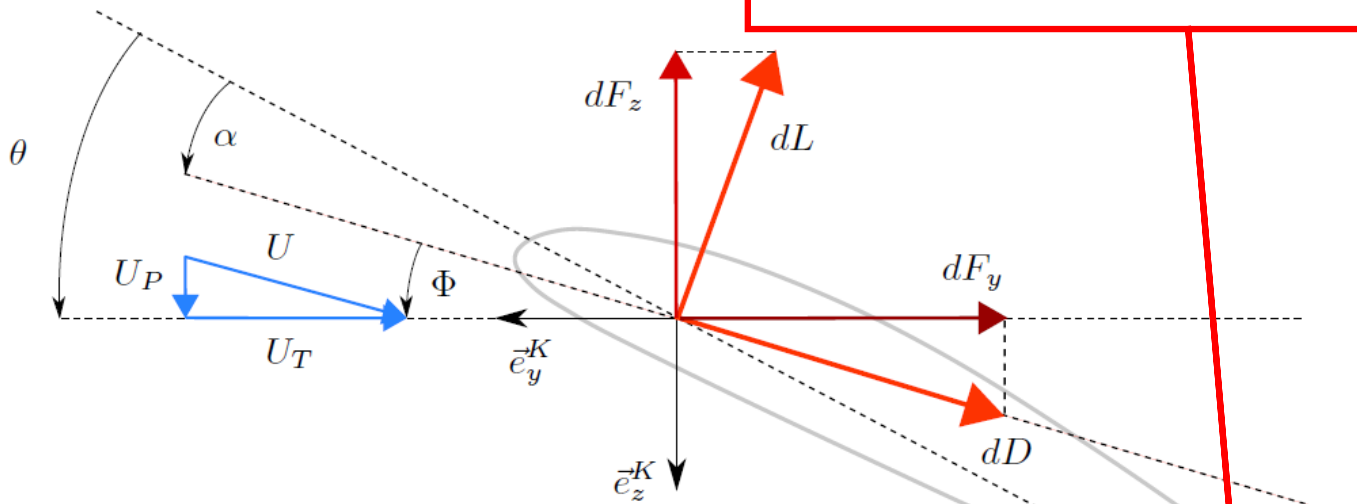




Rotorcraft Modeling: Rotor Feathering

- Blade Element Theory

$$\boxed{\theta(\xi) = \theta_0 + \theta_{1c}\,cos(\xi) + \theta_{1s}\,sin(\xi)}$$



$$dL = \frac{1}{2}\rho C_L U^2 c\,dr$$

$$dD = \frac{1}{2}\rho C_D U^2 c\,dr$$

$$\boxed{\begin{aligned} C_L &= C_L(\alpha, Re, Ma) \\ C_D &= C_D(\alpha, Re, Ma) \end{aligned}}$$

$$\alpha = \boxed{\theta} - \boxed{\phi}$$

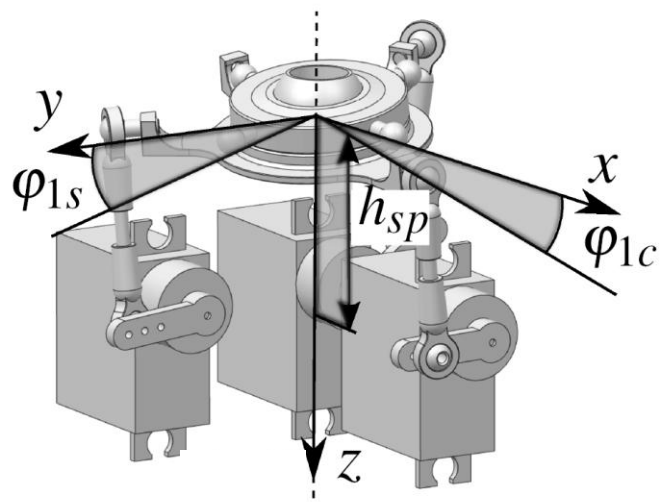
$$\boxed{\phi = atan(\frac{U_P}{U_T})}$$

$$\alpha = \theta - \boxed{\frac{U_P}{U_T}}$$

Rotorcraft Modeling: Rotor Feathering

- Pitch Actuation

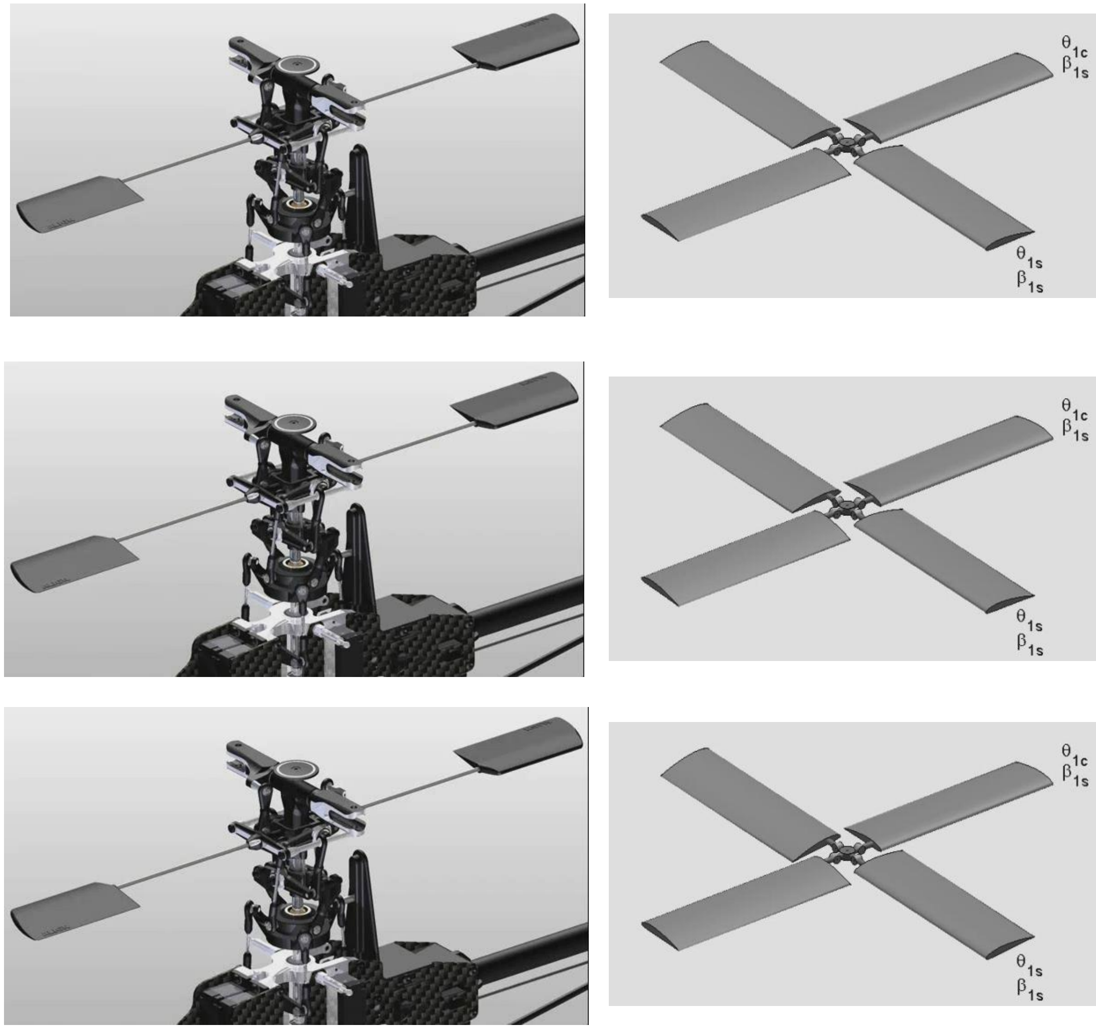
$$\boxed{\theta(\xi) = \theta_0 + \theta_{1c}\, cos(\xi) + \theta_{1s}\, sin(\xi)}$$



$$\theta_0^l = k_1^{cm}\, h_{sp} + k_2^{cm}$$

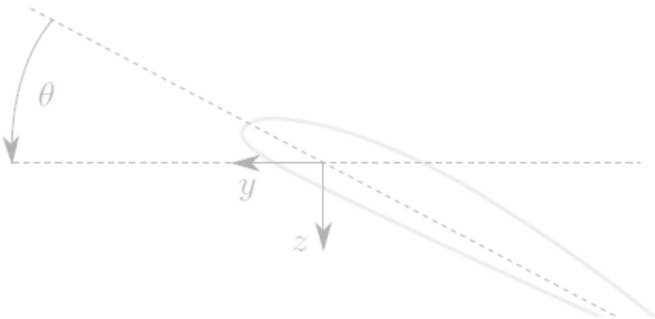
$$\theta_{1c}^l = k_3^{cm}\, \varphi_{1s} + k_4^{cm}\, \beta_{1s}^{fb}$$

$$\theta_{1s}^l = -k_3^{cm}\, \varphi_{1c} - k_4^{cm}\, \beta_{1c}^{fb}$$



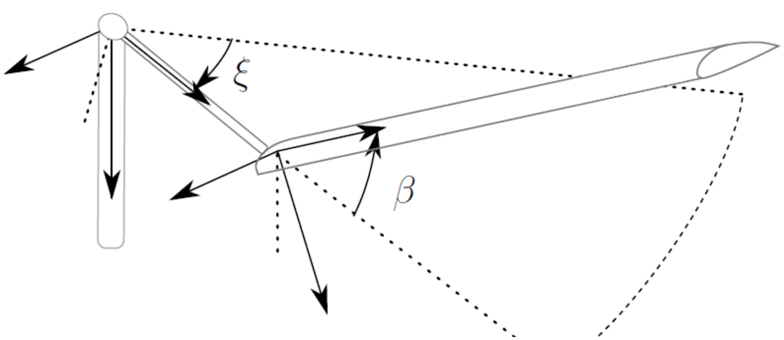
Rotorcraft Modeling: Rotor DOF

- Blade Feathering

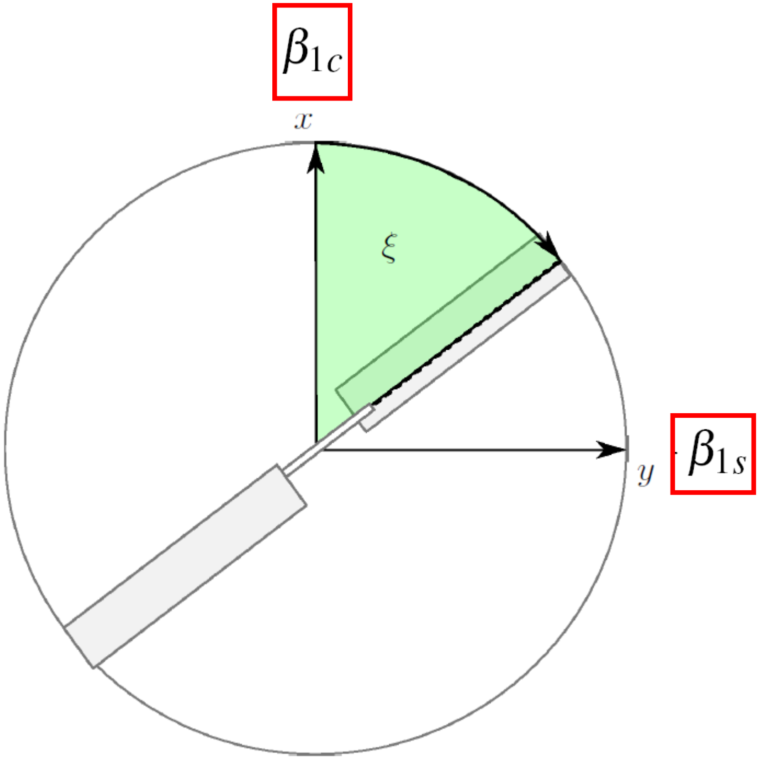


$$\theta(\xi) = \theta_0 + \theta_{1c}\, cos(\xi) + \theta_{1s}\, sin(\xi)$$

- Blade Flapping

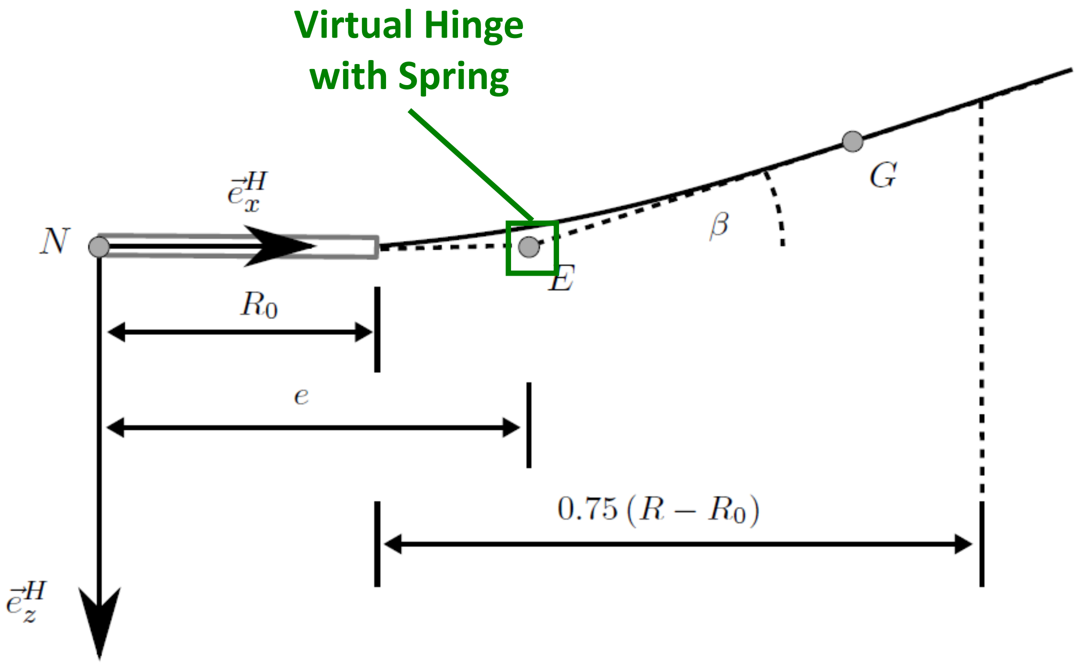


$$\beta(\xi) = \beta_0 + \beta_{1c}\, cos(\xi) + \beta_{1s}\, sin(\xi)$$



Rotorcraft Modeling: Rotor Flapping

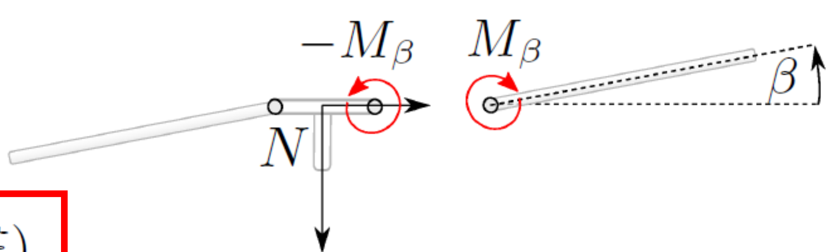
- Blade Flapping



$$M_\beta = -k_b\,\beta$$

Flap Moment

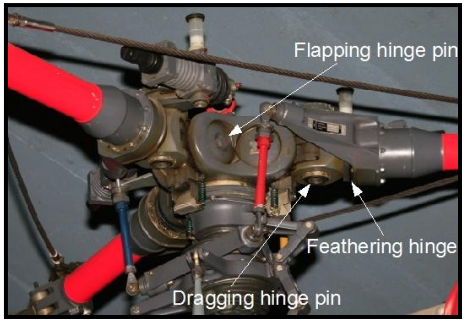
Couples Body and Rotor Dynamics

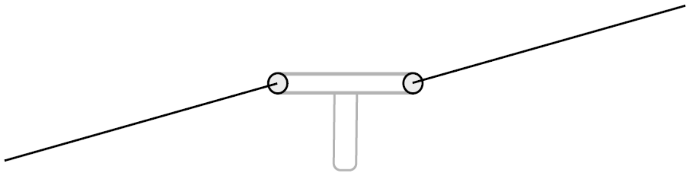


$$\beta(\xi) = \beta_0 + \beta_{1c}\cos(\xi) + \beta_{1s}\sin(\xi)$$

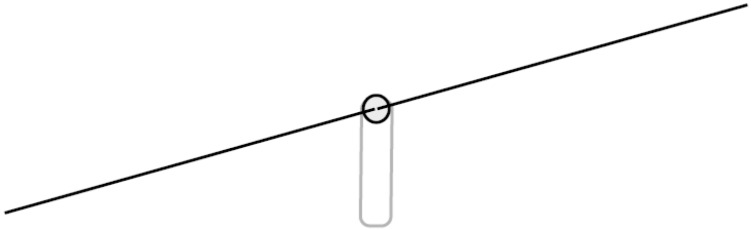
Rotorcraft Modeling: Rotor Flapping

- Fully Articulated

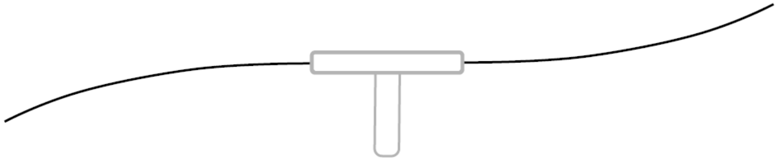




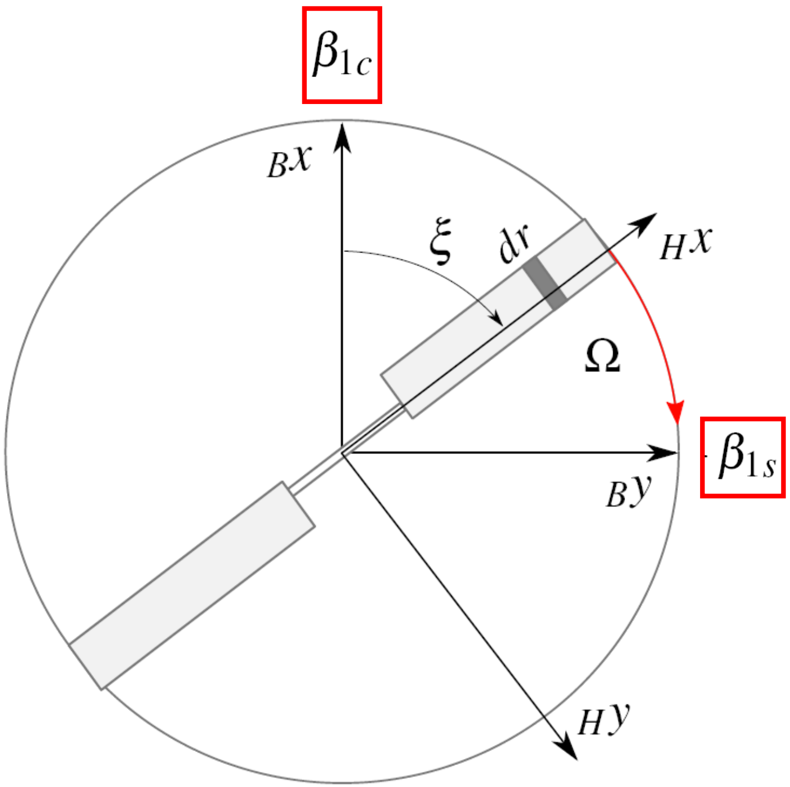
- Teetering

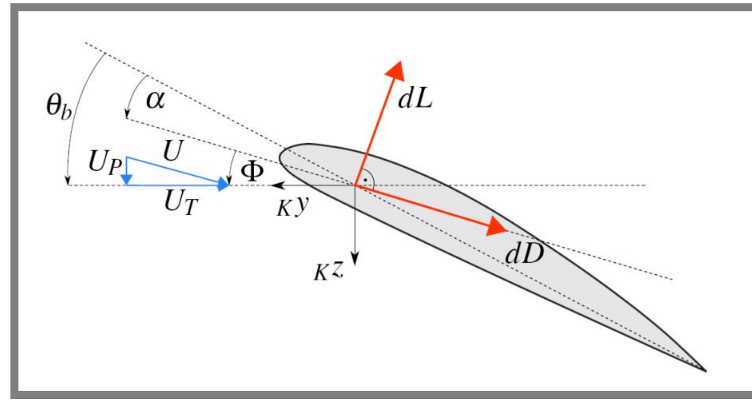


- Hingeless

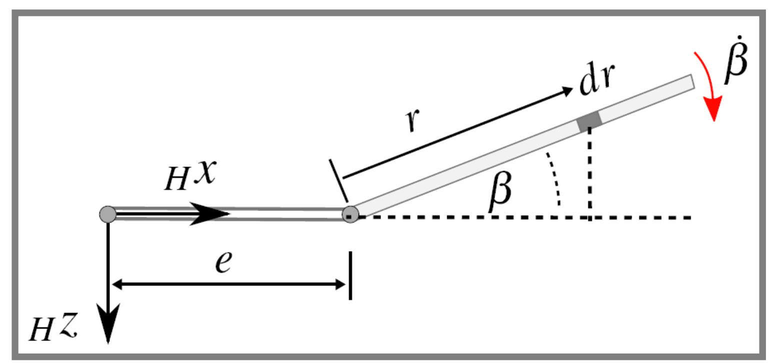


Rotorcraft Modeling: Flap Dynamics



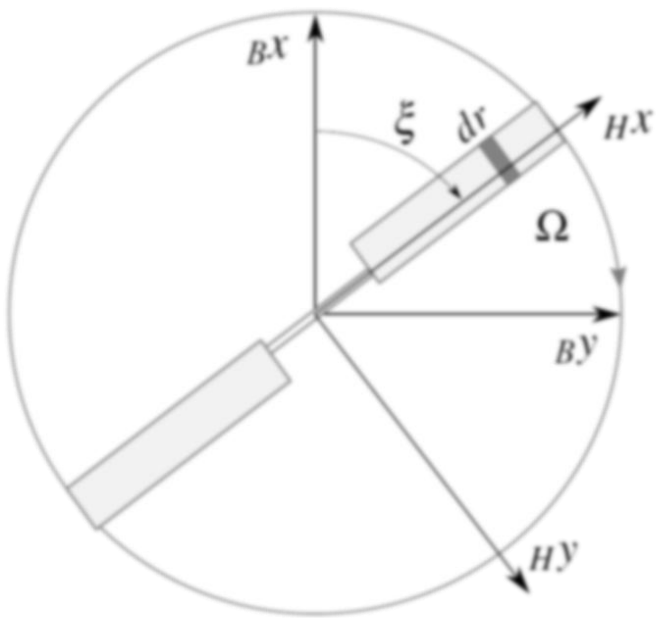


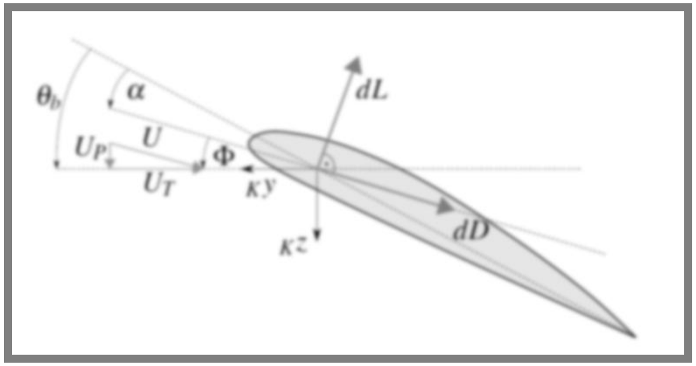
$$\theta_b(\xi) = \theta_0 + \theta_{1c}\cos(\xi) + \theta_{1s}\sin(\xi)$$

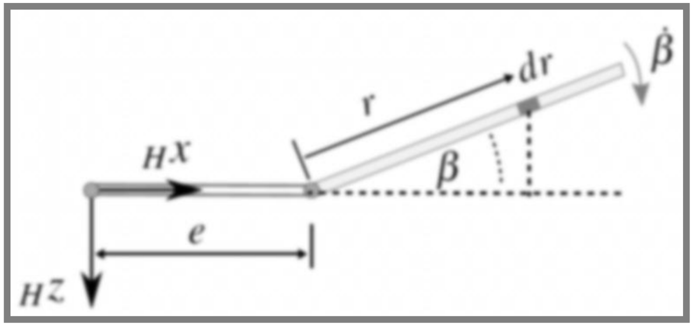


$$\beta(\xi) = \beta_0 + \beta_{1c}\,cos(\xi) + \beta_{1s}\,sin(\xi)$$

Rotorcraft Modeling: Flap Dynamics



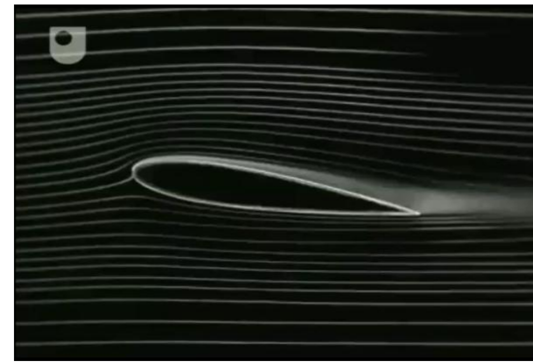




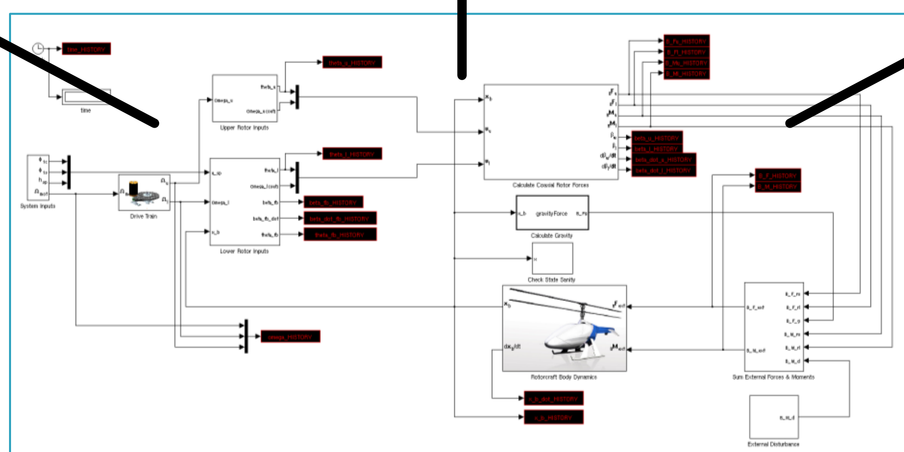
$$\dot{\beta}_{1c} = \Omega\left(\left(k_{1c} + \frac{k_{2c}}{\Omega^2}\right)\beta_{1c} + \left(k_{3c} + \frac{k_{4c}}{\Omega^2}\right)\beta_{1s} + k_{5c}\theta_{1c} + k_{6c}\theta_{1s} + k_{7c}\,\frac{q}{\Omega}\right)$$

$$\dot{\beta}_{1s} = \Omega\left(\left(k_{1s} + \frac{k_{2s}}{\Omega^2}\right)\beta_{1s} + \left(k_{3s} + \frac{k_{4s}}{\Omega^2}\right)\beta_{1c}\right) + k_{5s}\theta_{1s} + k_{6s}\theta_{1c} + k_{7s}\,\frac{p}{\Omega}$$

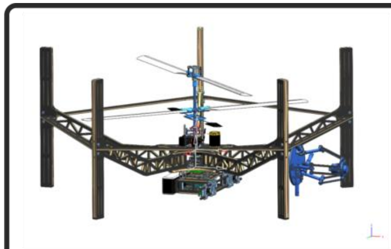
Rotorcraft Modeling: Assembling the System



Main Body Motion **Rotorblade Motion** **Rotorblade Aerodynamics**

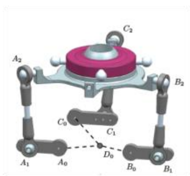


Rotorcraft Modeling: Assembling the System



18 States
6 of them not measurable
(but observable)
:: HYBRID MODEL ::

$$\dot{\vec{x}} = A\,\vec{x} + B\,\vec{u}$$



Swashplate

BLDC Motor

$$\vec{u} = \begin{pmatrix} \boxed{\begin{matrix} \varphi_{1c} \\ \varphi_{1s} \\ h_{sp} \end{matrix}} \\ \boxed{\tilde{\Omega}_{mot}} \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} x \\ u \\ y \\ v \\ \boxed{\begin{matrix} \phi \\ p \\ \theta \\ q \end{matrix}} \\ \boxed{\begin{matrix} \alpha \\ \beta \\ \varepsilon \\ \zeta \\ \gamma \\ \delta \end{matrix}} \\ \boxed{\begin{matrix} z \\ w \\ \psi \\ r \end{matrix}} \end{pmatrix}$$

18 States
Only 12 measurable
60 unknown parameters to identify only based on flight data

$$A = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -u_u & 0 & 0 & 0 & 0 & 0 & 0 & -u_\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -v_v & 0 & 0 & v_\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -w_w & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -p_\beta & 0 & -p_\delta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_\alpha & 0 & q_\gamma & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_r & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_u & 0 & \alpha_v & 0 & 0 & 0 & \alpha_p & 0 & -\alpha_q & 0 & 0 & -\alpha_\alpha & \alpha_\beta & 0 & 0 & -\alpha_\varepsilon & -\alpha_\zeta \\
0 & \beta_u & 0 & \beta_v & 0 & 0 & 0 & \beta_p & 0 & \beta_q & 0 & 0 & -\beta_\alpha & -\beta_\beta & 0 & 0 & \beta_\varepsilon & -\beta_\zeta \\
0 & \gamma_u & 0 & \gamma_v & 0 & 0 & 0 & -\gamma_p & 0 & -\gamma_q & 0 & 0 & 0 & 0 & -\gamma_\gamma & -\gamma_\delta & 0 & 0 \\
0 & \delta_u & 0 & \delta_v & 0 & 0 & 0 & \delta_p & 0 & -\delta_q & 0 & 0 & 0 & 0 & \delta_\gamma & -\delta_\delta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_q & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_\varepsilon & -\varepsilon_\zeta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \zeta_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \zeta_\varepsilon & -\zeta_\zeta
\end{pmatrix}, \quad \vec{x} = \begin{pmatrix} x \\ u \\ y \\ v \\ z \\ w \\ \phi \\ p \\ \theta \\ q \\ \psi \\ r \\ \alpha \\ \beta \\ \gamma \\ \delta \\ \varepsilon \\ \zeta \end{pmatrix}$$

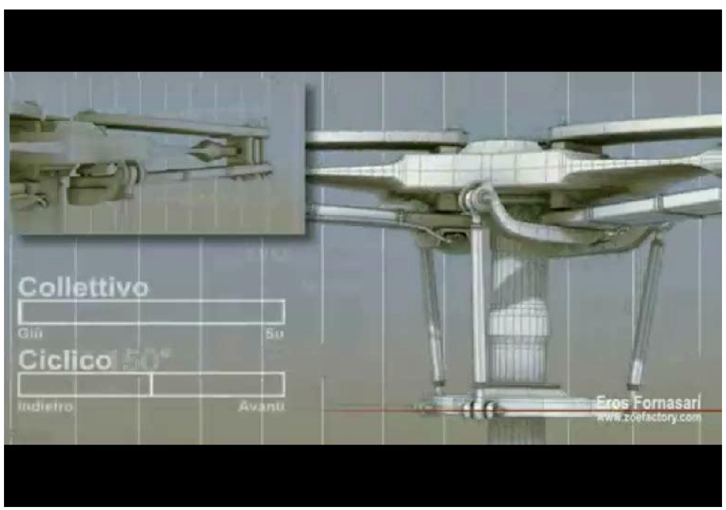
$$B = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -w_{h_{sp}} & -w_{\tilde{\Omega}_{mot}} \\
0 & 0 & 0 & 0 \\
0 & 0 & -p_{h_{sp}} & -p_{\tilde{\Omega}_{mot}} \\
0 & 0 & 0 & 0 \\
0 & 0 & q_{h_{sp}} & q_{\tilde{\Omega}_{mot}} \\
0 & 0 & 0 & 0 \\
0 & 0 & -r_{h_{sp}} & -r_{\tilde{\Omega}_{mot}} \\
-\alpha_{\varphi_{1c}} & -\alpha_{\varphi_{1s}} & 0 & 0 \\
\beta_{\varphi_{1c}} & -\beta_{\varphi_{1s}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-\varepsilon_{\varphi_{1c}} & -\varepsilon_{\varphi_{1s}} & 0 & 0 \\
\zeta_{\varphi_{1c}} & -\zeta_{\varphi_{1s}} & 0 & 0
\end{pmatrix}, \quad \vec{u} = \begin{pmatrix} \varphi_{1c} \\ \varphi_{1s} \\ h_{sp} \\ \tilde{\Omega}_{mot} \end{pmatrix}$$

Motivation

"A helicopter is a collection of oscillations held together by differential equations"

↓

White-Box models are often inaccurate or very difficult and time consuming to obtain



Blade flapping, coning and lagging increase the order of the system with states that are not directly measurable

Rotorcraft System Identification

Parametric System Identification

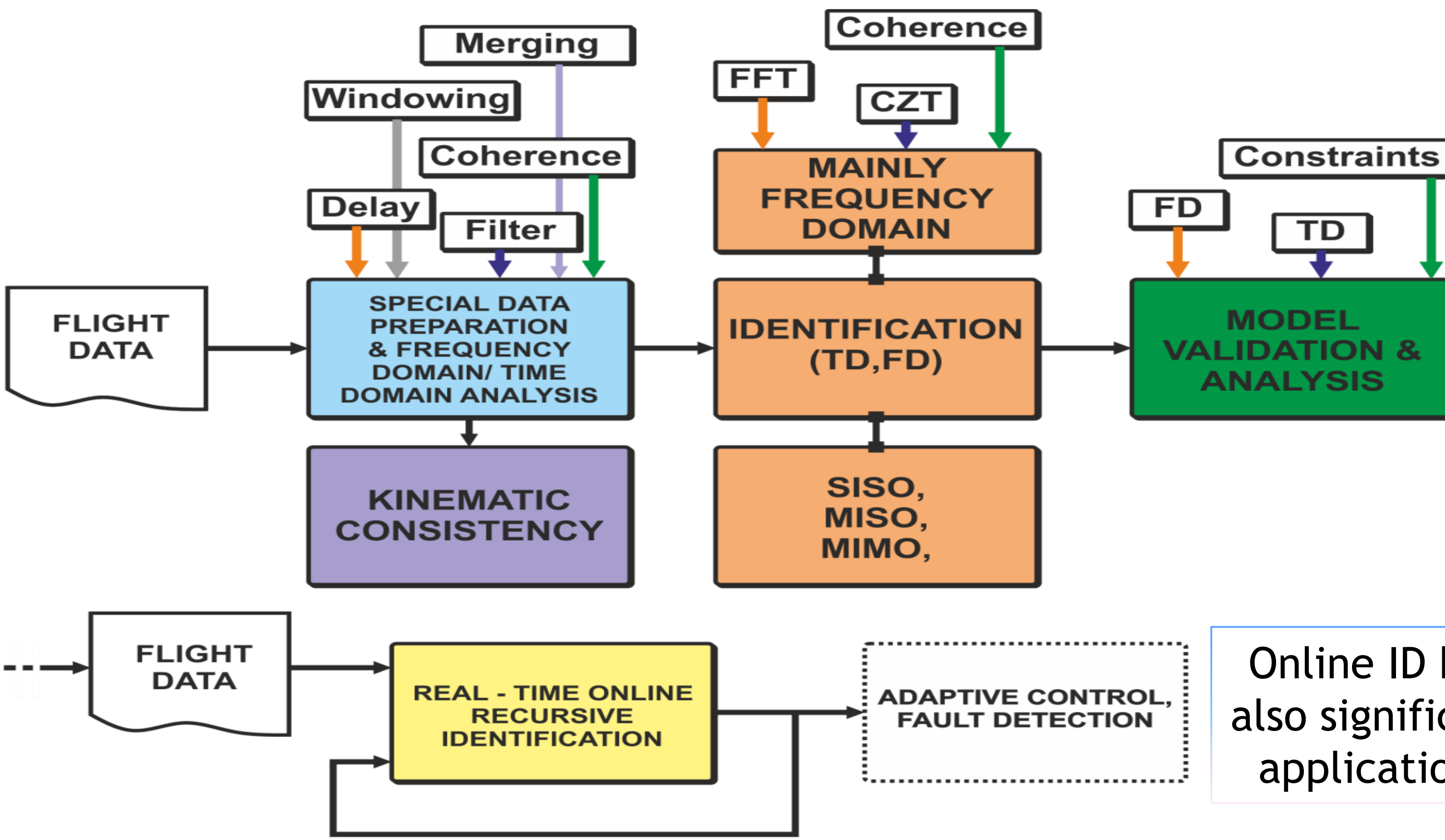
- Grey-Box
- Black-Box

- Frequency Domain Linear System Identification
- Time Domain Linear System Identification
- Frequency Domain Nonlinear System Identification
- Time Domain Nonlinear System Identification
- Hybrid Systems Identification

- Gradient Methods
- Evolutionary Methods

System Identification Steps

Record Proper Flight Data → Data Preparation → System Identification → Model Validation



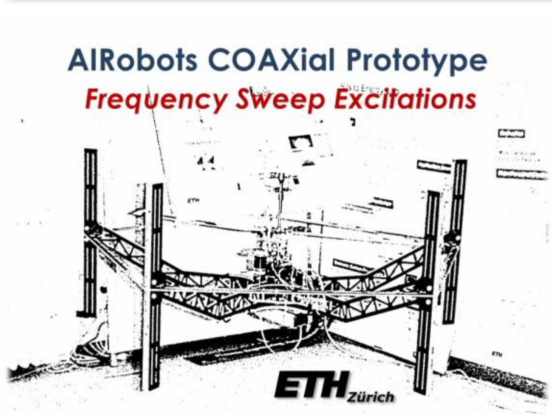
Online ID has also significant applications

Autonomous Systems Lab

Flight Experiments and Data Preparation

Record Proper Flight Data → Data Preparation – Use of empirical Metrics!

- Excite expected rotorcraft frequencies
- Flight long enough to capture low frequencies
- Start and end at trim
- Use Chrip Signals

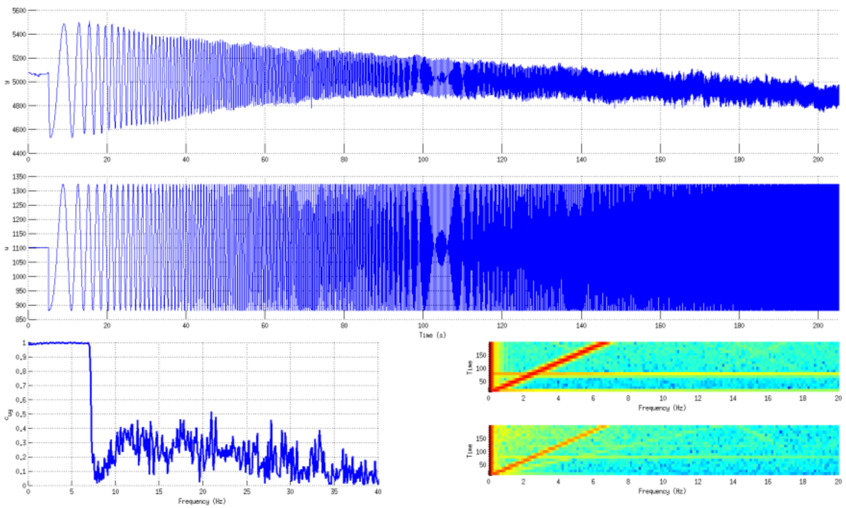


Detrend - Unbias	Based on trim
Estimate Input Delays	Actuation delays
Coherence (≥ 0.6) & Random Error Check	Check Linearity of the system
Persistence of Excitation Check	Conclusions about the order
I/O Spectrogram & Power Spectral Density	Visual understanding
Windowing	Increase accuracy over specific frequencies
Filter	

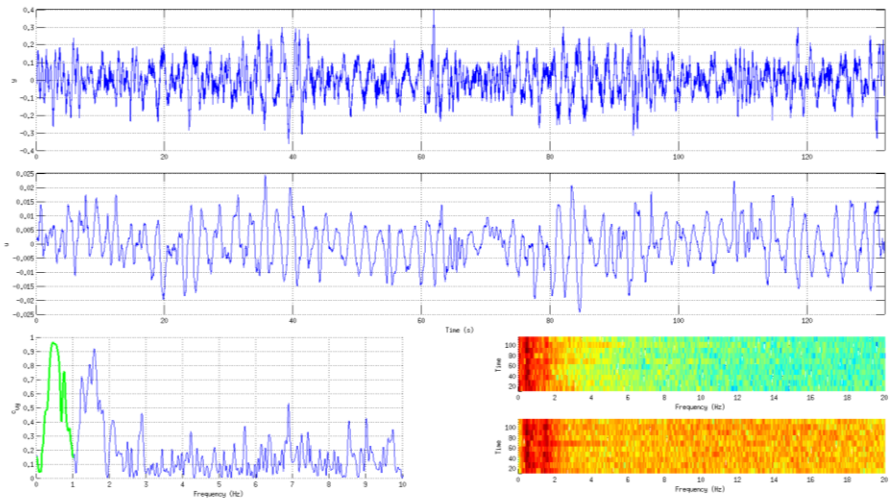
ETH Zürich

Flight Experiments and Data Preparation

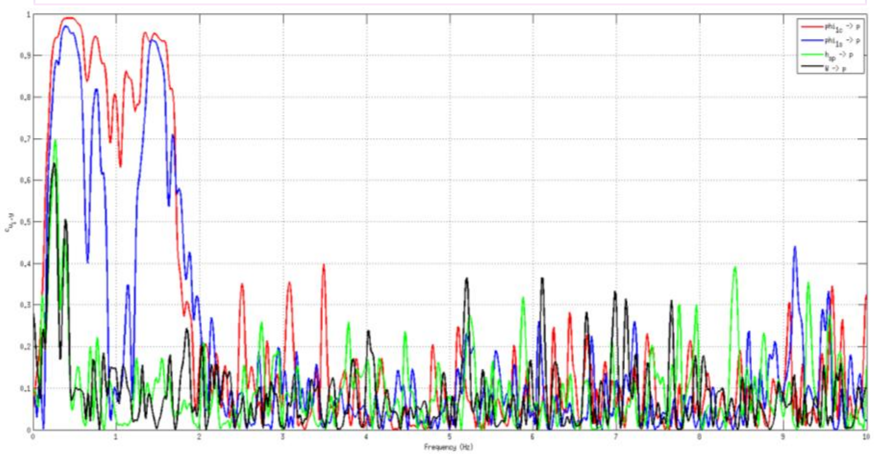
DC-Brushless Motor



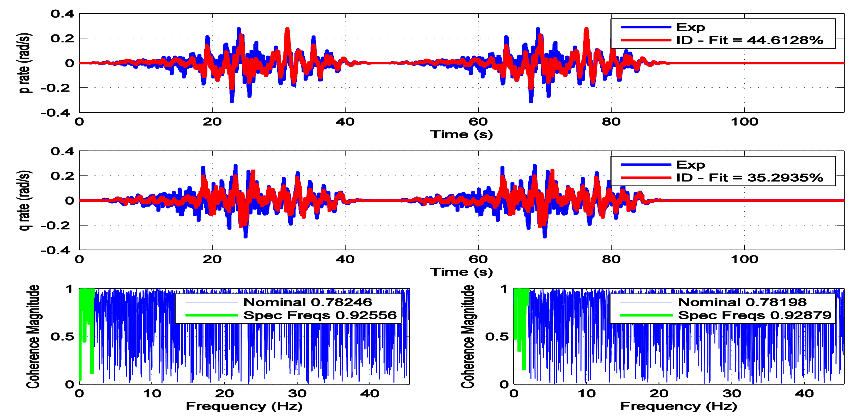
AIROBOTS Coaxial Prototype



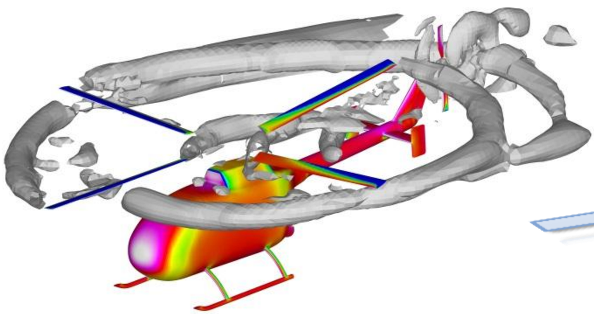
Use coherence to check Input/Output relations



Effect of windowing



Grey-box Model Derivation



$\dot{x} = f(x, u, \theta)$
x→rigid body and rotor states
u →swashplate and motor inputs
θ →vector of parameters to be identified

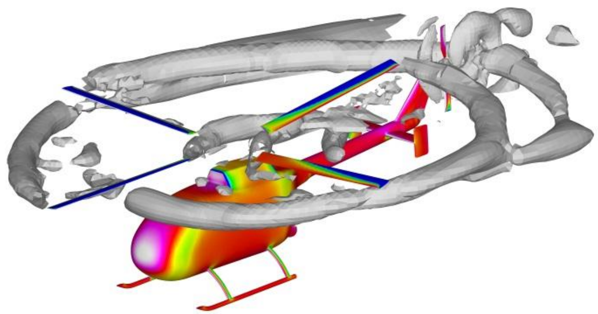
Define constraints for the vector of predicted parameters θ

Decide the level of accuracy and neglect some phenomena (i.e. blade lag)

Write the Nonlinear Differential Equations in Symbolic Form

Trim and Linearize the system around an operation point (i.e. hover, forward flight)

System Identification



$$\dot{x} = f(x, u, \theta)$$

Prepared Flight Data over specific frequencies and for specific degrees of freedom

Define Frequency Response Transform

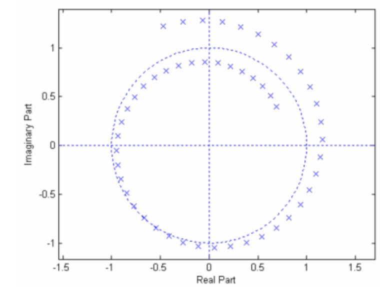
Fast Fourier Transform

Chirp-Z Transform

Computes the Z-Transform of a signal along spiral contours in the z-plane:

$$CZT(x[n]) = \sum_{n=0}^{N-1} x[n]\, z_k^{-n}$$

$$z_k = AW^{-k}, k = 0, \ldots, M-1$$



Define Objective Function

$$\mathbf{J} = \sum_{l=1}^{n_{TF}} \sum_{\omega_1}^{\omega_{n\omega}} W_\gamma(\omega_i)[W_g(|\hat{\mathbf{T}}_c(\omega_i)| - |\mathbf{T}(\omega_i)|)^2 + W_p(\angle\hat{\mathbf{T}}(\omega_i)_c - \angle\mathbf{T}(\omega_i))^2],$$

$$W_\gamma(\omega) = [1.58(1 - \exp^{-\gamma^2_{u_i, y_j}})]^2, \quad W_g = 1.0, \quad W_p = 0.01745,$$

Define Optimization Strategy

Solve constrained problem

Model Validation

Identified Model

Dataset not used in Identification

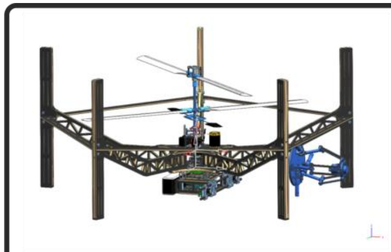
Model Validation

Check Fitting	Time & Frequency
Check Coherence	$[F_{LOW}, F_{HIGH}]$
I/O Spectrogram & Power Spectral Density	$[F_{LOW}, F_{HIGH}]$

Acceptable Fit?
Satisfy constraints?

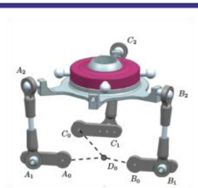
Initialize the model and run previous steps

AIROBOTS Coaxial Prototype Identification



18 States
6 of them not measurable
(but observable)
:: HYBRID MODEL ::

$$\dot{\vec{x}} = A\,\vec{x} + B\,\vec{u}$$



Swashplate

BLDC Motor

$$\vec{u} = \begin{pmatrix} \varphi_{1c} \\ \varphi_{1s} \\ h_{sp} \\ \tilde{\Omega}_{mot} \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} x \\ u \\ y \\ v \\ \phi \\ p \\ \theta \\ q \\ \alpha \\ \beta \\ \varepsilon \\ \zeta \\ \gamma \\ \delta \\ z \\ w \\ \psi \\ r \end{pmatrix}$$

18 States
Only 12 measurable
60 unknown parameters to
identify only based on flight data

$$A = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -u_u & 0 & 0 & 0 & 0 & 0 & 0 & -u_\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -v_v & 0 & 0 & v_\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -w_w & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -p_\beta & 0 & -p_\delta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_\alpha & 0 & q_\gamma & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_r & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_u & 0 & \alpha_v & 0 & 0 & 0 & \alpha_p & 0 & -\alpha_q & 0 & 0 & -\alpha_\alpha & \alpha_\beta & 0 & 0 & -\alpha_\varepsilon & -\alpha_\zeta \\
0 & \beta_u & 0 & \beta_v & 0 & 0 & 0 & \beta_p & 0 & \beta_q & 0 & 0 & -\beta_\alpha & -\beta_\beta & 0 & 0 & \beta_\varepsilon & -\beta_\zeta \\
0 & \gamma_u & 0 & \gamma_v & 0 & 0 & 0 & -\gamma_p & 0 & -\gamma_q & 0 & 0 & 0 & 0 & -\gamma_\gamma & -\gamma_\delta & 0 & 0 \\
0 & \delta_u & 0 & \delta_v & 0 & 0 & 0 & \delta_p & 0 & -\delta_q & 0 & 0 & 0 & 0 & \delta_\gamma & -\delta_\delta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_q & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_\varepsilon & -\varepsilon_\zeta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \zeta_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \zeta_\varepsilon & -\zeta_\zeta
\end{pmatrix}, \quad \vec{x} = \begin{pmatrix} x \\ u \\ y \\ v \\ z \\ w \\ \phi \\ p \\ \theta \\ q \\ \psi \\ r \\ \alpha \\ \beta \\ \gamma \\ \delta \\ \varepsilon \\ \zeta \end{pmatrix}$$

$$B = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -w_{h_{sp}} & -w_{\tilde{\Omega}_{mot}} \\
0 & 0 & 0 & 0 \\
0 & 0 & -p_{h_{sp}} & -p_{\tilde{\Omega}_{mot}} \\
0 & 0 & 0 & 0 \\
0 & 0 & q_{h_{sp}} & q_{\tilde{\Omega}_{mot}} \\
0 & 0 & 0 & 0 \\
0 & 0 & -r_{h_{sp}} & -r_{\tilde{\Omega}_{mot}} \\
-\alpha_{\varphi_{1c}} & -\alpha_{\varphi_{1s}} & 0 & 0 \\
\beta_{\varphi_{1c}} & -\beta_{\varphi_{1s}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-\varepsilon_{\varphi_{1c}} & -\varepsilon_{\varphi_{1s}} & 0 & 0 \\
\zeta_{\varphi_{1c}} & -\zeta_{\varphi_{1s}} & 0 & 0
\end{pmatrix}, \quad \vec{u} = \begin{pmatrix} \varphi_{1c} \\ \varphi_{1s} \\ h_{sp} \\ \tilde{\Omega}_{mot} \end{pmatrix}$$

AIROBOTS Coaxial Prototype Identification

Frequency Area

$[0.01,2]Hz$

Minimum Length

$[100 - 120]sec$

Lower "capturable" frequency $\approx 0.05Hz$

Frequency Response

Fast Fourier Transform

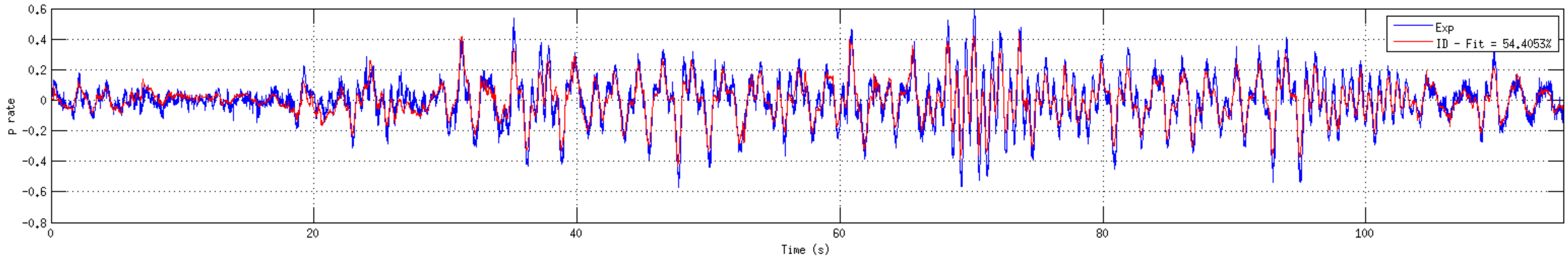
Chirp Z Transform

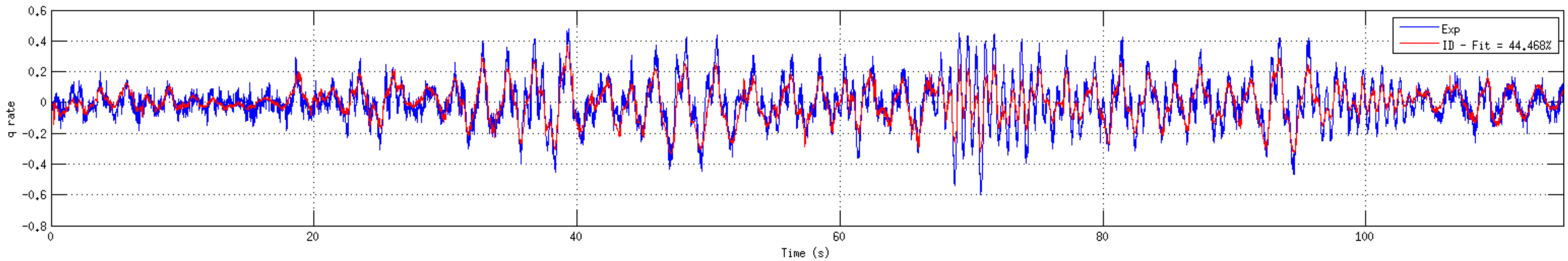
Optimization Algorithm

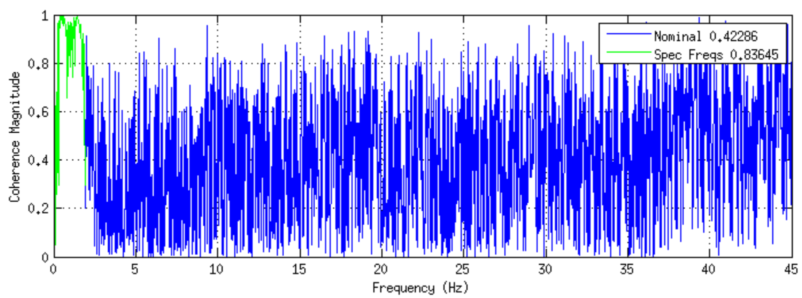
$$\begin{aligned} \mathbf{J} &= \sum_{l=1}^{n_{TF}} \sum_{\omega_1}^{\omega_{n\omega}} W_\gamma(\omega_i)[W_g(|\hat{\mathbf{T}}_c(\omega_i)| - |\mathbf{T}(\omega_i)|)^2 + W_p(\angle\hat{\mathbf{T}}(\omega_i)_c - \angle\mathbf{T}(\omega_i))^2], \\ W_\gamma(\omega) &= [1.58(1 - \exp^{-\gamma^2_{u_i,y_j}})]^2, \quad W_g = 1.0, \quad W_p = 0.01745, \end{aligned}$$

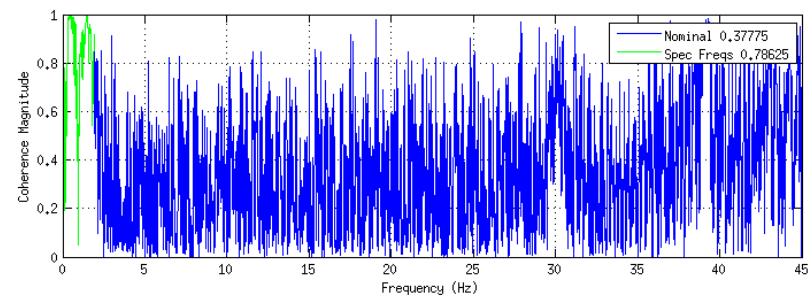
Mathworks MATLAB® classic gradient and adaptive gradient method

Roll rate (p), Pitch rate (q) Identification

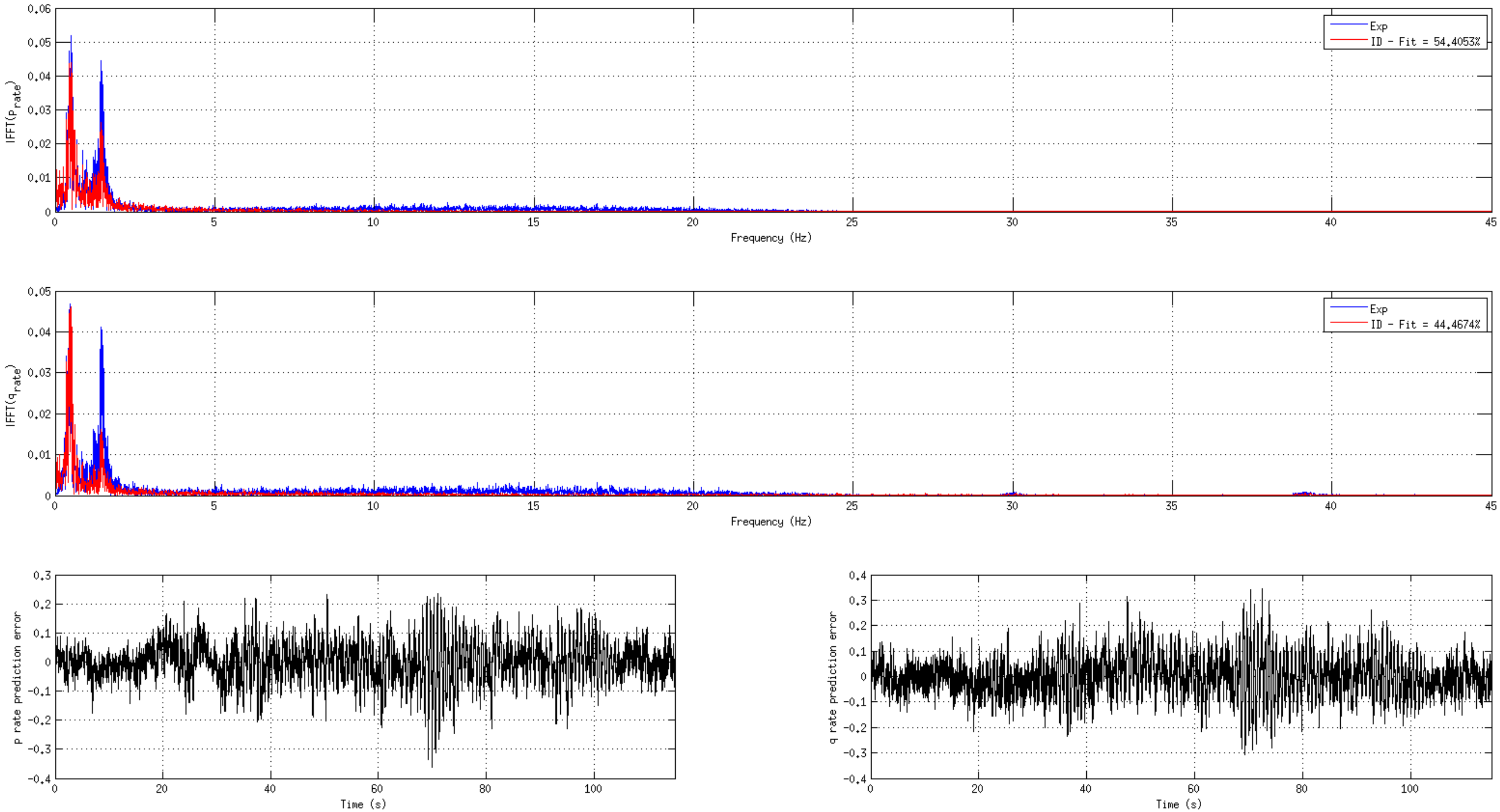




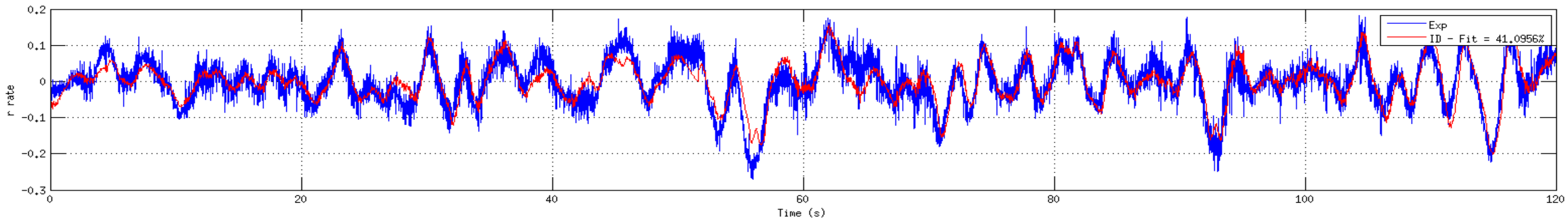


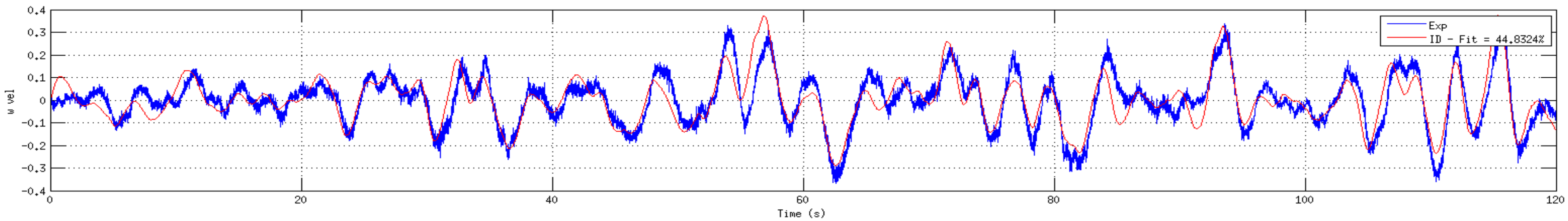


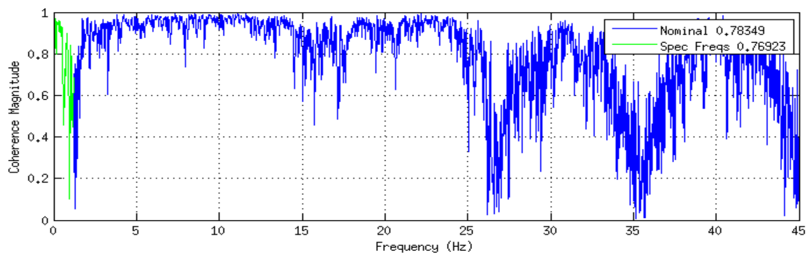
Roll rate (p), Pitch rate (q) Identification

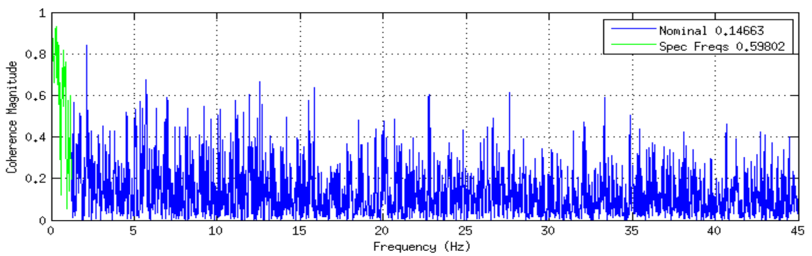


yaw rate (r), heave velocity(w) Identification







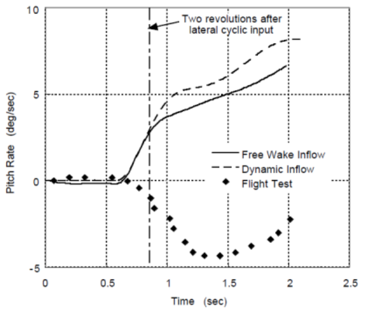


Autonomous Systems Lab

Identification as a multi-tool

- Accurate grey-box physically close models
- Simplified (Quasi-Steady) Models for Control Computation.
- Identify partial response of the system such as off-axis responses.
- Identify Closed-loop system response as a step for higher-level control (closed-loop attitude for velocity, velocity for trajectory control etc).
- Identify actuator dynamics as part of the selection process

How u_{roll} excites pitch?



System Identification is a research field but also a tool for the system and control engineer

Nonparametric Identification

Estimate the input/output relation of a system without a model but rather using the recorded data to nonparametrically calculate the frequency response of the system

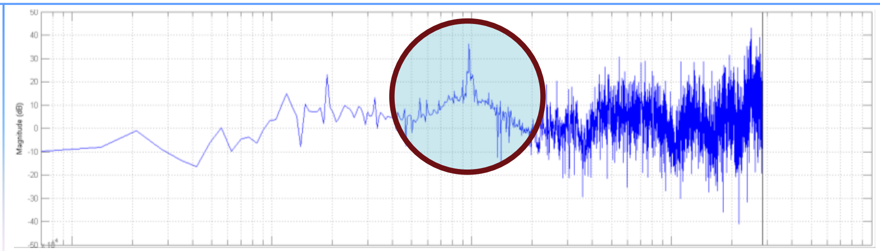
- Empirical Transfer Function

- Estimate Frequency Response with Fixed Frequency Resolution using Spectral Analysis

- Estimate Frequency Response and Spectrum using analysis with Frequency-dependent Resolution.

- Use of harmonic Windowing

Example application: Identify the Resonance frequency of the coupled rotors/fuselage dynamics



Autonomous Systems Lab

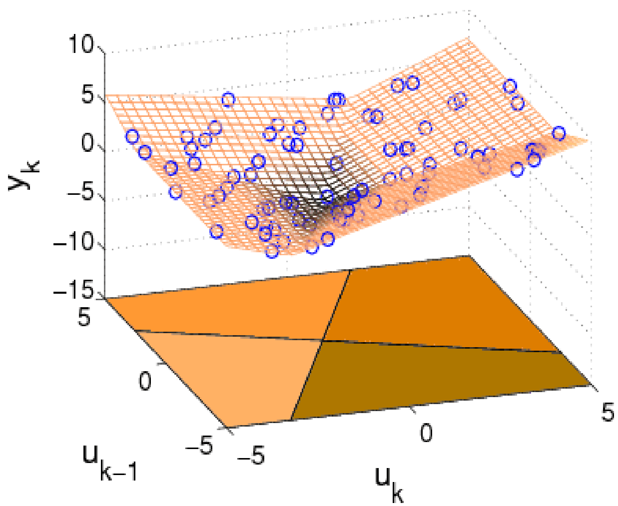
Research Trend

Nonlinear Frequency Domain System Identification:

- Possibly through the utilization of the generalized frequency response functions to reconstruct the model.
- Nonlinear Identification is until now dominated from time-domain approaches which lack the advantages of frequency-domain identification.

Hybrid Systems Identification:

- Hybrid systems often appear in robotics either due to physical interaction or due to modeling approach.
- Identify Piecewise Affine systems
  - Hinging-Hyperplane AutoRegressive eXogenous models (HHARX)
  - PieceWise affine AutoRegressive eXogenous models (PWARX)



ETH Zürich

Conclusions

- Grey-box System Identification can lead to accurate models that preserve the physicality of the system.
- Frequency-domain System Identification poses significant advantages for rotorcraft identification.
- All four main steps, flight experiments, data preparation, identification and model validation require special attention.
- The coupled rotors/fuselage model represents a special and challenging identification problem.
- Identification can be used as a tool in order to aid in various problems.
- Nonparametric identification can be very if properly used.
- Robotics can be benefitted from the aerospace community experience.